STEP Preparation

STEP is not a harder A-level paper. It is a different exercise: a small number of long questions, each building from a concrete case to a general result, where the marks are for the argument you write rather than the answer you reach.

The STEP at a glance

- **What it is:** the Sixth Term Examination Paper, used by Cambridge and Warwick as part of a conditional offer for mathematics, and accepted by several other universities.
- **Which papers:** STEP 2 and STEP 3; STEP 1 was discontinued in 2020. Which one you need is in your offer.
- **Here:** one STEP 2 question with its full mark scheme, to mark your own attempt against, and a preparation plan counting back from the sitting. Our STEP mock papers are being rewritten and are not published yet.
- **Dates, deadlines and which course needs what:** these change between cycles, so check the official specification rather than any summary of it.

What the STEP actually tests

Few questions, long answers

A STEP question is long enough to be a session's work on its own, and you attempt only some of the paper. That makes choosing which questions to attempt a skill in itself, and one almost nobody practises.

The parts are a ladder

An early part almost always exists to make a later one reachable. If part three looks impossible, the intended route runs through what you proved in part one.

Marked on the argument

Examiners read the reasoning. An unjustified step loses marks even when it happens to lead somewhere true, and a well-argued partial attempt scores better than most candidates expect.

Four things that decide the grade, and none of them is knowing more mathematics

Finish fewer, properly

A complete, well-argued answer to a small number of questions beats fragments of many. Most candidates lose more to abandoning questions half way than to being unable to do them.

Read every part first

The last part usually tells you what the first part was for. A question that looks impossible at part three is normally reachable through what part one handed you, and reading forwards changes what you do with the opening.

Justify, do not assert

An unjustified step loses marks even when it leads somewhere true. Saying which earlier result you are using, and why a step is valid, is where a large share of the marks live.

Choose before you commit

Deciding which questions to attempt is a skill, and almost nobody practises it. Read the parts rather than the topic headings: a familiar topic with a nasty final part is a worse bet than an unfamiliar one that ladders cleanly.

Learn to mark your own proof

The hardest part of preparing alone is that there is no answer to check against, and the model solution is only one correct write-up out of many. Mark against what a solution has to establish instead, which is what the mark scheme below is for.

Mark your own attempt

What a candidate preparing alone lacks is not a score. It is a way of telling whether the thing they wrote would have earned the marks. This exercise is that: one question, then your own write-up scored against a written mark scheme.

1. Work the question below on paper, with no clock for the first attempt and nothing else open. Write it as though someone will read it, because that is the thing being assessed.
2. Stop when you stop. A partial attempt is worth marking, and marking one honestly is more useful than finishing by looking something up.
3. Only then open the mark scheme. For each result it names, decide two separate things: did you reach it, and did you justify it. They are different, and the gap between them is the STEP-specific skill.
4. Score yourself with the table below, then compare the shape of your argument with the results the mark scheme names, since the model solution is not published at the moment.

5. Write down the single step you failed to justify. That sentence is worth more than the score.

Merging two fractions into one

This is a proof question, so there is no answer key to compute. The model solution is not published at the moment.

Difficulty 10 of 18, STEP 2 core. This is the most approachable question in the STEP 2 mock paper it was written for, chosen because meeting the format for the first time is enough to be getting on with.

In this question n denotes an integer with $n \geq 2$.

1. **(i)** [4 marks]

   Let x and y be positive real numbers and let p and q be real numbers. Find, in its simplest form, a single expression for
   $$\frac{p^2}{x} + \frac{q^2}{y} - \frac{(p+q)^2}{x+y},$$
   and hence show that
   $$\frac{p^2}{x} + \frac{q^2}{y} \geq \frac{(p+q)^2}{x+y}.$$
   Determine precisely the values of p, q, x and y for which equality holds.

2. **(ii)** [6 marks]

   Let $p_1, p_2, \ldots, p_n$ be real numbers and let $x_1, x_2, \ldots, x_n$ be positive real numbers. Prove by induction on n that
   $$\frac{p_1^2}{x_1} + \frac{p_2^2}{x_2} + \cdots + \frac{p_n^2}{x_n} \geq \frac{(p_1 + p_2 + \cdots + p_n)^2}{x_1 + x_2 + \cdots + x_n},$$
   and that equality holds if and only if
   $$\frac{p_1}{x_1} = \frac{p_2}{x_2} = \cdots = \frac{p_n}{x_n}.$$

3. **(iii)** [4 marks]

   Let $a_1, a_2, \ldots, a_n$ be positive real numbers, and let a_{n+1} denote a_1. Show that
   $$\frac{a_1^2}{a_2} + \frac{a_2^2}{a_3} + \cdots + \frac{a_n^2}{a_{n+1}} \geq a_1 + a_2 + \cdots + a_n,$$
   and determine precisely when equality holds.

4. **(iv)** [6 marks]

   Let $a_1, a_2, \ldots, a_n$ be positive real numbers and write $S = a_1 + a_2 + \cdots + a_n$. Show that
   $$\frac{a_1}{S - a_1} + \frac{a_2}{S - a_2} + \cdots + \frac{a_n}{S - a_n} \geq \frac{n}{n-1},$$

and determine precisely when equality holds.

Open the mark scheme. Write your attempt first.

For each line below, mark yourself twice: **R** if you reached it, and **J** if your write-up justifies it well enough that a marker would not have to fill anything in. An R without a J is the characteristic STEP loss.

(i) [4 marks]

- The identity
$$\frac{p^2}{x} + \frac{q^2}{y} - \frac{(p+q)^2}{x+y} = \frac{(py-qx)^2}{xy(x+y)}$$
, valid for all real p and q and all positive x and y.
- $xy(x+y) > 0$ and $(py-qx)^2 \geq 0$, hence $\frac{p^2}{x} + \frac{q^2}{y} \geq \frac{(p+q)^2}{x+y}$.
- Equality holds if and only if $py = qx$, equivalently $\frac{p}{x} = \frac{q}{y}$, and both directions are established.
- **2 marks:** Combining the three fractions over $xy(x+y)$ and simplifying the numerator to $(py-qx)^2$ or to an equivalent perfect square. One of these two marks is for the correct expansion; the other is for recognising the numerator as a square rather than leaving it as $p^2y^2 - 2pqxy + q^2x^2$.
- **1 mark:** Deducing the inequality, with the positivity of the denominator stated. A solution that multiplies through by $xy(x+y)$ instead earns this mark only if it says that this quantity is positive.
- **1 mark:** The equality condition $py = qx$, argued in both directions: that $py = qx$ gives equality, and that equality forces $py = qx$ because the denominator cannot vanish.
- **No credit:** Multiplying the inequality through by $xy(x+y)$ without stating that this quantity is positive. The direction of the resulting inequality is then unjustified, and this is the standard way to lose the deduction mark.
- **No credit:** Quoting the Cauchy-Schwarz inequality, or naming this result as a known lemma, in place of a proof. The question asks for the difference in its simplest form, so the algebra is the required work.
- **No credit:** Verifying the inequality at particular values of p, q, x, y, or arguing from a sketch. Neither establishes a statement quantified over all real p and q.

(ii) [6 marks]

- A stated proposition $S(n)$ quantified over all real $p_1, \ldots, p_n$ and all positive $x_1, \ldots, x_n$, carrying the equality criterion as part of the statement.
- The base case: $S(2)$ is part (i).
- With $P_m = p_1 + \cdots + p_m$ and $X_m = x_1 + \cdots + x_m$, the inductive step inequality
$$\sum_{i=1}^{n+1} \frac{p_i^2}{x_i} \geq \frac{P_n^2}{X_n} + \frac{p_{n+1}^2}{x_{n+1}} \geq \frac{P_{n+1}^2}{X_{n+1}}$$
, with the first inequality attributed to $S(n)$ and the second to part (i) applied with $p = P_n$, $q = p_{n+1}$, $x = X_n$, $y = x_{n+1}$, and with $X_n > 0$ noted.
- Equality in $S(n+1)$ holds if and only if it holds at both inequalities of the chain.
- If $\frac{p_1}{x_1} = \cdots = \frac{p_{n+1}}{x_{n+1}} = k$ then $P_n = kX_n$, so both inequalities are equalities.

- Conversely, equality forces $\frac{p_1}{x_1} = \cdots = \frac{p_n}{x_n} = k$ by $S(n)$, hence $\frac{P_n}{X_n} = k$, and then part (i) forces $\frac{p_{n+1}}{x_{n+1}} = \frac{P_n}{X_n} = k$.
- **1 mark:** A correctly stated proposition, quantified over all admissible lists of length n and including the equality clause, together with a base case: $S(2)$ identified as part (i), or $S(1)$ with its equality clause checked.
- **2 marks:** The two-step chain of the inductive step, with each inequality attributed to the result that justifies it and with the positivity of X_n stated where part (i) is invoked.
- **1 mark:** The direction: equal ratios imply equality. The mark requires the intermediate observation $P_n = kX_n$, not just an assertion.
- **2 marks:** The converse direction: equality implies equal ratios. One mark for using the equality clause of the inductive hypothesis to get the first n ratios equal, one for the deduction $\frac{p_{n+1}}{x_{n+1}} = \frac{P_n}{X_n} = k$ from the equality case of part (i).
- **No credit:** Proving the inequality and asserting the equality criterion. The criterion carries half the marks in this part, and the converse direction is the half that is not routine.
- **No credit:** An inductive hypothesis stated for one fixed list of n numbers. The step applies the hypothesis to the first n entries of a longer list, so the hypothesis has to be quantified over all lists of that length.
- **No credit:** An inductive step written as a chain of equivalences starting from the statement to be proved and ending at something true, with no remark that the steps are reversible.
- **No credit:** Omitting the base case, or checking it at $n = 1$ for the inequality alone. A base case at $n = 1$ is valid: with one term both sides equal $\frac{p_1^2}{x_1}$, so equality always holds and a single ratio meets the equal-ratio condition trivially, and the step from $n = 1$ to $n = 2$ is exactly part (i). It earns the mark only if the write-up says how the equality clause reads for one term.

(iii) [4 marks]

- The choice $p_i = a_i$, $x_i = a_{i+1}$ with $a_{n+1} = a_1$, and the observation that $x_1 + \cdots + x_n = a_1 + \cdots + a_n$ because the denominators are a rearrangement of the numerators' bases.
- Part (ii) then gives $\sum_{i=1}^{n} \frac{a_i^2}{a_{i+1}} \geq \frac{(a_1 + \cdots + a_n)^2}{a_1 + \cdots + a_n} = a_1 + \cdots + a_n$.
- Equality holds if and only if $\frac{a_1}{a_2} = \frac{a_2}{a_3} = \cdots = \frac{a_n}{a_1}$.
- Writing that common value as k, the product of the n equal fractions is 1, so $k^n = 1$; since $k > 0$ this gives $k = 1$ and hence $a_1 = \cdots = a_n$. The converse is checked.
- **1 mark:** Choosing the cyclic weights and stating why the sum of the denominators equals the sum of the a_i.
- **1 mark:** The resulting inequality, correctly quoted from part (ii).
- **2 marks:** The equality argument. One mark for reducing to the equal-ratio condition and forming the product of the n ratios; one for $k^n = 1$ with $k > 0$ giving $k = 1$, and for checking the converse.
- **No credit:** Deducing $a_1 = \cdots = a_n$ directly from the equal-ratio condition without using the ratio that closes the cycle. Equality of $\frac{a_1}{a_2}, \ldots, \frac{a_{n-1}}{a_n}$ alone says only that the a_i form a geometric progression, which does not force them to be equal.
- **No credit:** Taking $k^n = 1$ to give $k = 1$ without noting that k is positive. Over the reals with n even, $k = -1$ also satisfies $k^n = 1$.
- **No credit:** Applying part (ii) with $x_i = a_i$, which yields $a_1 + \cdots + a_n$ on both sides and proves nothing about the stated sum.

(iv) [6 marks]

- $S - a_i > 0$ for every i, because $n \geq 2$ and every a_j is positive.
- The construction $\frac{a_i}{S-a_i} = \frac{a_i^2}{a_i(S-a_i)}$ with $a_i(S-a_i) > 0$.
- $\sum_{i=1}^{n} a_i(S-a_i) = S^2 - Q$ where $Q = a_1^2 + \cdots + a_n^2$, and $S^2 - Q > 0$.
- By part (ii), $\sum_{i=1}^{n} \frac{a_i}{S-a_i} \geq \frac{S^2}{S^2-Q}$.
- $Q \geq \frac{S^2}{n}$, obtained from part (ii) with $p_i = a_i$ and $x_i = 1$.
- $S^2 - Q \leq \frac{(n-1)S^2}{n}$, both sides positive, hence $\frac{S^2}{S^2-Q} \geq \frac{n}{n-1}$.
- Equality holds if and only if $a_1 = a_2 = \cdots = a_n$, established in both directions.
- **2 marks:** The construction $\frac{a_i}{S-a_i} = \frac{a_i^2}{a_i(S-a_i)}$, which puts each term in the form (square) over (positive), and the application of part (ii) to it. One mark for the construction, one for the correct resulting bound.
- **1 mark:** Evaluating $\sum_i a_i(S-a_i)$ as $S^2 - Q$ and stating that this is positive.
- **2 marks:** The lower bound $Q \geq \frac{S^2}{n}$ and the correctly directed deduction to $\frac{n}{n-1}$. One mark for obtaining the bound on Q, one for turning a lower bound on Q into a lower bound on $\frac{S^2}{S^2-Q}$ with the reversal explained.
- **1 mark:** Equality precisely when all the a_i are equal, with both the forward deduction from the two equality criteria and the verification that equal values give $\frac{n}{n-1}$.
- **No credit:** Writing $\frac{a_i}{S-a_i} = \frac{(\sqrt{a_i})^2}{S-a_i}$ and applying part (ii) to reach $\frac{(\sqrt{a_1} + \cdots + \sqrt{a_n})^2}{(n-1)S}$. The step is valid but the bound is too weak to finish: closing it would need $(\sum \sqrt{a_i})^2 \geq nS$, and part (ii) with $p_i = \sqrt{a_i}$, $x_i = 1$ gives the opposite inequality. Work that stops here earns none of the marks for this part, although the positivity of the denominators still counts as a result reached.
- **No credit:** Bounding Q above rather than below. An upper bound for Q gives an upper bound for $\frac{S^2}{S^2-Q}$, which is the wrong direction and settles nothing.
- **No credit:** Assuming $S = 1$ without saying why that is permitted. The claim is homogeneous of degree 0 in the a_i, so the normalisation is legitimate, but the sentence justifying it has to be there.
- **No credit:** Verifying the inequality only at $a_1 = \cdots = a_n$ and concluding that this is the minimum.

Reading your own score

Count the results you reached, and separately the ones you justified. This question names 20 of them. The pattern matters more than the total.

What you found	What it means	What to do about it
Reached few, justified few	The mathematics is the bottleneck, not the write-up.	`Work the method library e`
Reached most, justified few	The most common result, and the	`Rewrite this attempt prop`

What you found	What it means	What to do about it
	one STEP punishes hardest. You can do the mathematics and are not yet writing it as an argument.	
Reached and justified the early parts, stalled later	The ladder did its job and you climbed off it. Usually the later part needed an earlier result you did not think to reuse.	`Look at what the earlier`
Reached and justified nearly all	You are working at the level this question was written for.	`Move to a harder question`

This is a self-assessment, not a grade. It cannot tell you what you would score in the exam itself, and nothing on this site claims otherwise.

The model solution

When it is published, read it after you have marked yourself, and compare the shape of the argument rather than the wording. If you proved the same things by a different route, you have not made a mistake: that is what it looks like when a proof question is working properly.

The mock paper this exercise was written around is being rebuilt, so its worked solutions are not published at the moment. Our ESAT worked solutions show the same standard of write-up on questions that are published in full.

Why there is no STEP diagnostic

A diagnostic that scores multiple-choice questions works for the ESAT. Building one for STEP would be straightforward and would be a mistake. STEP has no options to choose between, and the marks are for the argument rather than the answer, so a multiple-choice adaptation would measure exactly the part of the subject STEP does not test. What a candidate preparing alone actually lacks is not a score. It is a way of telling whether the thing they wrote would have earned the marks.

The plan

Seven phases. Each one names what you are trying to achieve, what to work through, and how to tell when you are done with it. The check at the end of a phase is the part worth taking seriously: it is what stops you moving on because the weeks have passed rather than because the work is done.

This plan counts back from an assumed sitting on 3 June 2027. That date is our working assumption, not an announcement: check the official STEP page for the date in your cycle and shift the phases to match.

1. Find out what you are actually being asked to do

   STEP is not a harder A-level paper, and most of the preparation candidates do first is preparation for the wrong exam. Read what the paper rewards, then read the shapes of argument it asks for, before working a single question.

   - What the STEP tests
   - The STEP Method Library

   Done when you can say, in one sentence each, what an unjustified step costs you and why the parts of a question are a ladder.

2. Work one paper untimed, and mark it yourself

   Take one question and give it as long as it needs, with no clock and no solution in front of you. Then mark your own write-up against the mark scheme. The gap between what you reached and what you justified is what the rest of the plan is for.

   - Mark your own STEP attempt

   Done when you have marked one full attempt of your own and can name the specific step you failed to justify.

3. Build the techniques, one family at a time

   Work through the method families rather than through the papers. When a question defeats you, find its family in the library, read the entry, and go back to the question rather than to the solution.

   - The STEP Method Library

   Done when you can name the family a question belongs to before you know how to do it.

4. STEP 3, if your offer names it

   The STEP 3 papers assume the full Further Mathematics content and lean harder on sustained technical work. Skip this phase only if your offer does not name STEP 3: check the offer rather than assuming.

   Done when a STEP 3 question takes you the same effort a STEP 2 one did at the start of the plan.

5. Practise choosing, not just solving

   Deciding which questions to attempt is itself a skill, and it is the one nobody practises. Read a whole paper, pick your questions from the parts rather than from the topics, and commit. Then check whether the ones you skipped were actually the harder ones.

   Done when your choice of questions, made in the first few minutes, matches the ones you would choose knowing the answers.

6. Write-ups, and only write-ups

   At this point the mathematics is what it is, and the remaining marks are in how you write. Take questions you have already solved and write them again properly: state what you are doing and why, name the earlier result each part consumes, and handle the equality case and the degenerate case explicitly.

   - Mark your own STEP attempt
   - The STEP Method Library

   Done when a marker could follow your solution without reconstructing anything, and when you stop losing marks you had already earned.

7. Stop adding, start consolidating

   Nothing new. Re-read your own marked attempts and the reservations printed on the questions you found hardest. A complete, well-argued answer to a small number of questions beats fragments of many, and that is decided by nerve rather than by knowledge.

   - What the STEP tests

   Done when you are choosing questions calmly and finishing the ones you start.

- **The date is an assumption.** The plan counts back from one assumed sitting date; check the official one for your cycle and treat the windows as approximate.
- **Start late without panicking.** If you are arriving part way through, the phase marked *you are here* is where to pick up. Work forward from today rather than trying to catch up on everything before it.
- **Which paper you need is in your offer.** Read the offer rather than assuming, and check the official specification for what each paper assumes. Those requirements change between cycles and no tutor's summary should be trusted over the source.

Common questions

- **Which STEP papers are actually sat?** STEP 2 and STEP 3. STEP 1 was discontinued in 2020, so a resource built around it is modelling a paper that no longer exists. Everything published here is written against STEP 2 or STEP 3 for that reason. Which one your offer names is in the offer, and the official specification is the place to check what each assumes.
- **Is STEP just a harder A-level paper?** No, and treating it as one is the common mistake. STEP asks for a small number of long questions where the marks are for the argument you write. A complete, well-written answer to two questions beats fragments of five.
- **How do you choose which questions to attempt?** Read all the parts of a question before starting, not just the first. The last part usually reveals what the first part was for, and a question that looks impossible at part three is often reachable through what you proved at part one.
- **Are marks for the answer or the working?** The working. An unjustified step loses marks even when it leads somewhere true, and a well-argued partial attempt scores better than most candidates expect.
- **How do you mark your own proof when there is no answer to check against?** Against what a solution has to establish, rather than against how someone else phrased it: three correct write-ups of one proposition are three different pieces of prose. The question on this page comes with a written mark scheme naming the intermediate results a solution must reach, and the self-marking exercise walks you through scoring one of your own attempts against it.
- **Are there practice papers on this site for STEP?** Not at the moment. The mock papers are being rewritten to the same standard as our ESAT papers, and nothing is published until they meet it; the one question on this page is the exception. The official papers from past years remain the final word on the exam itself either way.

How this STEP material was checked

The question on this page, its mark scheme and the plan were written by us. The marking procedure is a reasonable approximation of how a long-form paper is marked, not a reproduction of any official scheme, and the plan's sequence is a judgement about how to spend the months before a sitting, not a measured result. How we check material in general is on our how we work page.

Use it to organise your preparation and as extra practice, alongside the official papers from past years, which remain the final word on the exam itself. Found an error? Tell us at hello@mintnib.com and it will be fixed or withdrawn.

Sitting more than one of these?

Candidates often sit two admissions tests in the same cycle, and the overlap is larger than it looks: the pure mathematics is shared, and the habits that earn marks in one paper usually earn them in another. Each page below covers what its test rewards and what free practice exists for it.

- TMUA: Test of Mathematics for University Admission Two papers and Multiple choice and Pure mathematics

- MAT: Mathematics Admissions Test Retired after 2025 and Oxford now uses the TMUA
- PAT: Physics Aptitude Test Retired after October 2025 and Oxford now uses the ESAT
- ESAT: Engineering and Science Admissions Test The deepest library we publish