Coffee and milk: when should you add it?

A classic interview question, answered in full. In an ordinary mug, adding the milk immediately leaves the coffee about 2.5 °C hotter after ten minutes than adding the same milk at the ten-minute mark. This page explains why, level by level as an interviewer's follow-ups add complications, how each variable increases or decreases that difference, which kind of mug makes adding the milk at the end the better choice, and how to present the answer in an interview.

The question, as an interviewer asks it

"You have just made a cup of coffee, but you will not drink it for ten minutes. You take milk. When should you add the milk, if you want the coffee as hot as possible when you drink it?"

That is the whole question, and it is deliberately incomplete. Nobody has said how much milk, how cold it is, how warm the room is, what shape the mug is, or whether anyone blows on it. Part of what the interviewer is watching is whether you notice.

Before you calculate: ask, or assume out loud

A strong answer starts with the missing information, either as questions or as stated assumptions. Each one changes the answer by an amount this page computes with its full model (level 4), for fridge milk in a straight mug unless the line says otherwise:

- **How much milk?** The amount changes the result more than any other variable: with 5 ml of milk the order changes the final temperature by 0.5 °C, with 120 ml by 6.1 °C.
- **How cold is the milk?** Milk from the fridge increases the advantage of adding it immediately, from 2.0 °C with milk at room temperature to 2.5 °C. Milk kept hotter than about 69 °C until it is used makes adding it at the end the better choice.
- **How warm is the room?** From a room at 10 °C to one at 30 °C, the final temperature with the milk added immediately increases from 64.1 to 67.1 °C, and Δ only from 2.5 to 2.7 °C.
- **What shape is the mug?** In straight, tapered and bowl-shaped mugs, adding the milk immediately leaves the coffee hotter, by 1.8 to 3.2 °C; in a cup that flares out just above the coffee, adding it at the end leaves the coffee 0.5 °C hotter.

- **Is the air still?** Air moving over the cup fast enough to make the convection and evaporation coefficients 3 times their still-air values (about what blowing does) lowers the final temperature with the milk added immediately from 65.5 to 50.9 °C, and increases Δ from 2.5 to 3.4 °C.
- **Is the milk waiting in the fridge or on the table?** Milk left on the table warms to about 8 °C in ten minutes, which reduces the advantage to 2.3 °C.

Then commit to a base case and say it: **250 ml of coffee at 90 °C in a straight ceramic mug 8 cm across, 30 ml of milk from the fridge at 5 °C, a room at 20 °C with still air, and ten minutes to wait.** Students who raise air motion and mug shape without being asked are doing exactly what the question is for.

The short answer, and what it depends on

Add the milk immediately. In the base case, on the page's full model (level 4), the coffee is at 65.5 °C at minute ten with the milk added immediately, and at 63.0 °C with the same milk added at minute ten: 2.5 °C hotter. With milk at room temperature the difference is 2.0 °C.

On this page, **Δ** always means the temperature at minute ten with the milk added immediately minus the temperature with the milk added at minute ten. A positive Δ means adding the milk immediately leaves the coffee hotter.

Δ depends on the mug (with 30 ml of milk, in a cup that flares out just above the coffee, Δ is negative), on the amount of milk (in a straight mug, more milk gives a larger Δ), and on whether the milk is kept hot while you wait (above about 69 °C, Δ is negative). The ladder below builds the answer one follow-up question at a time, from the 30-second answer to the most complete model.

The ladder: from the 30-second answer to the full model

Each level opens with the follow-up question an interviewer asks next, adds one complication, and says what changed. Stop at the level you need.

1. Level 0: the 30-second answer: *So, when do you add the milk?*
2. Level 1: the coffee cools while you wait: *What if the coffee cools while you wait?*
3. Level 2: the milk is not at room temperature: *What if the milk comes from the fridge, or is kept hot?*
4. Level 3: where the heat actually goes: *Is convection the only way the coffee loses heat?*
5. Level 4: the leak per degree is not constant: *Does the coffee really lose the same fraction of its excess every minute?*
6. Level 5: the shape of the mug: *Does the shape of the mug matter?*
7. Level 6: the other variables, one at a time: *What else changes the answer, and by how much?*
8. Level 7: the cup left for hours: *What happens if nobody drinks it?*

After the ladder: what an interviewer is testing at each level; what students often get wrong; where students get stuck, and the step that unsticks it; more follow-up questions an interviewer may ask; check it against your kitchen; where this puzzle comes from.

Level 0: the 30-second answer

The follow-up. *"So, when do you add the milk?"*

The intuition. A hot drink loses heat faster than a warm one, because the rate of heat loss increases with the difference between the drink and the room. Adding cold milk immediately lowers that difference for the whole wait, so the drink loses less heat in total. That reasoning, said in two sentences, is a strong first answer.

The simplest model: mixing is a weighted average, and no heat is lost.

Mixing is a weighted average

No heat leaves during the instant of mixing, so the heat the coffee gives up is the heat the milk takes in. With $C = 1300$ J K^{-1} for coffee and mug, $c = 121$ J K^{-1} for the milk and T_m the milk's temperature:

$$C\,(T - T_{\text{mix}}) = c\,(T_{\text{mix}} - T_m)$$
$$\Rightarrow \quad T_{\text{mix}} = \frac{C\,T + c\,T_m}{C + c}$$

Fridge milk into coffee at 90 °C gives 82.8 °C. **Celsius and kelvin give the same answer here:** add 273.15 to both temperatures and the result increases by exactly 273.15, because the two weights add up to one. A weighted average works in any temperature scale. Kelvin is needed wherever a formula uses the temperature itself rather than a temperature difference or a weighted average whose weights add up to one: radiation's T^4 in level 3, and the film temperature T_{film} in the Rayleigh number (the assumptions of level 3).

The result at this level. If no heat is lost while you wait, both orders end at the same temperature, 82.8 °C: the coffee is still at 90 °C when the milk goes in, whenever that is. So the order can matter only through the heat lost during the wait, which is what every level below adds.

Assumptions added at this level, each with why it holds:

- **The milk mixes at once**, and no heat leaves during the mixing itself: pouring and stirring take seconds, against a wait of minutes.
- **The drink has one temperature.** Coffee cooled at the surface and the walls is denser and sinks, so the drink keeps stirring itself, and pouring the milk mixes it. The model does not compute differences inside the drink; it assumes they stay small next to the 70 °C between the drink and the room. It would fail for a layered drink that nobody stirs.

What changed from the level before. This is the starting point: with no heat loss, Δ is exactly zero.

Level 1: the coffee cools while you wait

The follow-up. *"What if the coffee cools while you wait?"*

Now heat leaks out, at a rate proportional to the difference between the drink and the room. This is Newton's law of cooling, derived in full below. To isolate the effect of the wait, this level uses milk at room temperature; level 2 adds the milk's own temperature.

Newton's law of cooling, derived in full

Step 1, energy. $T(t)$ is the drink's temperature at time t, starting at $T_0 = 90$ °C, in a room at $T_r = 20$ °C. Let C be the heat capacity of coffee and mug together: 1045 J K⁻¹ for the coffee plus 255 J K⁻¹ for the ceramic, $C = 1300$ J K⁻¹. The heat stored above a reference temperature T_{ref} is $C\,(T - T_{\text{ref}})$, so its rate of change is $C\,dT\,/\,dt$. If heat leaves at Q watts, energy conservation says the two are equal and opposite:

$$C\,\frac{dT}{dt} = -\,Q(T)$$

This is a differential equation: it gives the rate of change of T, not T itself. At 90 °C, $Q \approx 69$ W (level 3 shows where each watt goes), so the coffee starts by losing 3.2 °C a minute. The minus sign says the temperature decreases while heat leaves.

Step 2, the simplest leak. "Loss proportional to the gap" becomes $Q = K\,(T - T_r)$, where T_r is the room temperature and K, in W K⁻¹, measures the leakiness. For convection alone $K = hA$, with h the convection coefficient and A the area. In our mug at 90 °C, the four routes of level 3 together give $K = Q\,/\,(T - T_r) \approx 69\,/\,70 \approx 0.99$ W K⁻¹. This level treats K as one measured number; level 3 shows what it is made of. This is Newton's law of cooling: exact for a fixed coefficient, and a fair first approximation for any smooth loss over a small range of temperature.

Step 3, a substitution: measure from the room. Let $u = T - T_r$, the excess over the room. The room is fixed, so $du\,/\,dt = dT\,/\,dt$, and with $\tau = C\,/\,K$:

$$\frac{du}{dt} = -\,\frac{u}{\tau}$$

Why substitute: in T the right side is $-K(T - T_r)\,/\,C$, which carries the constant T_r; in u the rate is simply proportional to the unknown itself, and the next two steps take one line each. *Another way:* keep T and integrate $dT\,/\,(T - T_r)$ directly, which gives the same logarithm; or multiply by the integrating factor $e^{t\,/\,\tau}$. The substitution is a convenience, not a necessity.

Step 4, why the answer must be exponential. The equation says that in each small interval the gap loses the same *fraction* of itself: $1\,/\,\tau$ per unit time, whatever the gap is. In our mug that is about 4.5 % of the gap in the first minute. A quantity that loses a fixed fraction per unit time is an exponential, like radioactive decay or compound interest run backwards. Steps 5 and 6 prove it.

Step 5, separate the variables. The equation mixes u and t. Divide both sides by u, which is allowed because $u > 0$ throughout: in this model the coffee approaches the room but never reaches it.

$$\frac{1}{u}\,\frac{du}{dt} = -\,\frac{1}{\tau}$$

By the chain rule the left side is the derivative of $\ln u$ with respect to t. Both sides are now derivatives of something we know: $\frac{d}{dt}\ln u = \frac{d}{dt}\left(-\frac{t}{\tau}\right)$. *Why this step:* integrating undoes a derivative, but only when each side is a derivative of a known function; separating the variables arranges exactly that. Writing $du\,/\,u = -\,dt\,/\,\tau$ with differentials is shorthand for this chain-rule step.

Step 6, integrate from the start to time t. Integrate both sides from 0 to t; on the left the variable runs from the starting gap $u_0 = 70$ K to u:

$$\int_{u_0}^{u} \frac{du'}{u'} = -\frac{1}{\tau}\int_{0}^{t} dt'$$
$$\Rightarrow \quad \ln u - \ln u_0 = -\frac{t}{\tau}$$

Definite limits fix the constant of integration for us. With indefinite integrals you would write $\ln u = -t/\tau + k$ and find $k = \ln u_0$ by putting $t = 0$: the same thing. Take the exponential of both sides:

$$u = u_0\, e^{-t/\tau}, \qquad \tau = \frac{C}{K}$$

The constant of integration has become a multiplying factor, u_0, not an added constant. *Another way:* guess $u = a\, e^{\lambda t}$, substitute it into Step 3 to get $\lambda = -1/\tau$, and use the start to get $a = u_0$. Guessing works because Step 4 told us what to guess.

Step 7, read the time constant. τ is stored heat per kelvin over leak per kelvin: J K^{-1} divided by W K^{-1} is J W^{-1}, which is seconds. It is the time to lose all the excess heat at the starting rate; in that time the gap falls to $1/e \approx 37$ % of its starting value, after 2τ to 14 %, and it halves every $\tau\ln 2$. Ours is about 22 minutes at 90 °C, so the gap would halve in about 15 minutes.

Comparing the two orders: the general condition

Let K be the leak of the black coffee and K' the leak after the milk raises the level. Measure from the room, so the milk's excess is u_m (-15 K for fridge milk; $u_0 = 70$ K for the coffee). After a wait t, with the milk added immediately (first line) or at the end of the wait (second line):

$$u_{\text{imm}} = \frac{C\,u_0 + c\,u_m}{C + c}\, e^{-K' t/(C+c)}$$
$$u_{\text{end}} = \frac{C\,u_0\, e^{-Kt/C} + c\,u_m}{C + c}$$

With milk at room temperature ($u_m = 0$) each is the same constant, $C\,u_0/(C+c)$, times one exponential. Why divide one by the other: the shared constant cancels and the comparison becomes a single exponent whose sign decides it (subtracting works too, but leaves a difference of exponentials to compare). The ratio is

$$\frac{u_{\text{imm}}}{u_{\text{end}}} = \exp\left[t\left(\frac{K}{C} - \frac{K'}{C+c}\right)\right]$$

It is greater than 1 exactly when its exponent is positive, $K/C > K'/(C+c)$. Multiply both sides by C/K, which is positive: **adding the milk immediately leaves the coffee hotter exactly when**

$$\frac{K'}{K} < \frac{C+c}{C}$$

If one coefficient h applies to every surface, substitute $K = hA$: h cancels and the condition becomes pure geometry,

$$A_{\text{after}}/A_{\text{before}} < C_{\text{after}}/C_{\text{before}}$$

: the milk must add relatively more heat capacity than surface. Ignoring the mug and the small difference in specific heat, that is

$$A_{\text{after}}/A_{\text{before}} < m_{\text{after}}/m_{\text{before}}$$

. In our mug the milk adds 9.3 % of heat capacity and only 2.3 % of leak. With one h for every surface, our mug would only just pass: its total area increases by 8.8 % against 9.3 %. It passes by a wide margin because the wetted wall loses far less heat per square centimetre than the open top (the loss budget of level 3).

The result at this level. With milk at room temperature and K fixed at its 90 °C value, the coffee is at 61.8 °C at minute ten with the milk added immediately and 60.6 °C with it added at the end: Δ = 1.2 °C.

Assumptions added at this level, each with why it holds:

- **The leak is proportional to the gap**, with a fixed K. Level 4 removes this.
- **The mug is warm from the start.** 90 °C is the temperature just after pouring, with the mug already at the coffee's temperature: its 255 J K⁻¹ count as part of the drink's 1300 J K⁻¹. A cold mug takes its share of heat in the first moments; that lowers the starting point for both orders by the same amount, and the starting-temperature panels in level 6 show what a lower start does to Δ.
- **The room stays at 20 °C.** In ten minutes black coffee gives the room about 28 kJ. Spread through a room's air, walls and furniture, that is far too little to change its temperature. It would matter in a small closed space, such as a cool bag.

What changed from the level before. Heat now leaks while you wait, so the order matters. With milk at room temperature and Newton's law, Δ is positive exactly when the milk adds a larger fraction of heat capacity than of leak; in our mug Δ = 1.2 °C, against zero at level 0.

Level 2: the milk is not at room temperature

The follow-up. *"What if the milk comes from the fridge, or is kept hot?"*

For milk at any temperature, subtract the two formulas of level 1. To keep it short, write $E = e^{-Kt/C}$ and $E' = e^{-K't/(C+c)}$, the two decay factors, and multiply both sides by $C + c$:

$$(C + c)(u_{\text{imm}} - u_{\text{end}}) = C\,u_0\,(E' - E) + c\,u_m\,(E' - 1)$$

The first term is the room-temperature result of level 1. The second has $E' - 1 < 0$, so it is positive when $u_m < 0$ and negative when $u_m > 0$:

- **Fridge milk** ($u_m < 0$): with the milk added immediately, its deficit below the room decays along with the rest of the drink; with the milk added at the end, the whole deficit arrives at the last moment. So fridge milk increases Δ.
- **Milk kept hot** ($u_m > 0$): with the milk added immediately, its excess heat leaks out during the wait; added at the end, it arrives with nothing lost. So hot milk decreases Δ, and makes it negative above about 69 °C (full model).

That is also why a cup must flare further before adding the milk at the end is better with fridge milk (50 degrees, against 41 degrees with milk at room temperature; level 5).

The result at this level. With milk from the fridge at 5 °C and the same fixed K, the coffee is at 61.0 °C at minute ten with the milk added immediately and 59.3 °C with it added at the end: Δ = 1.7 °C, against 1.2 °C with milk at room temperature.

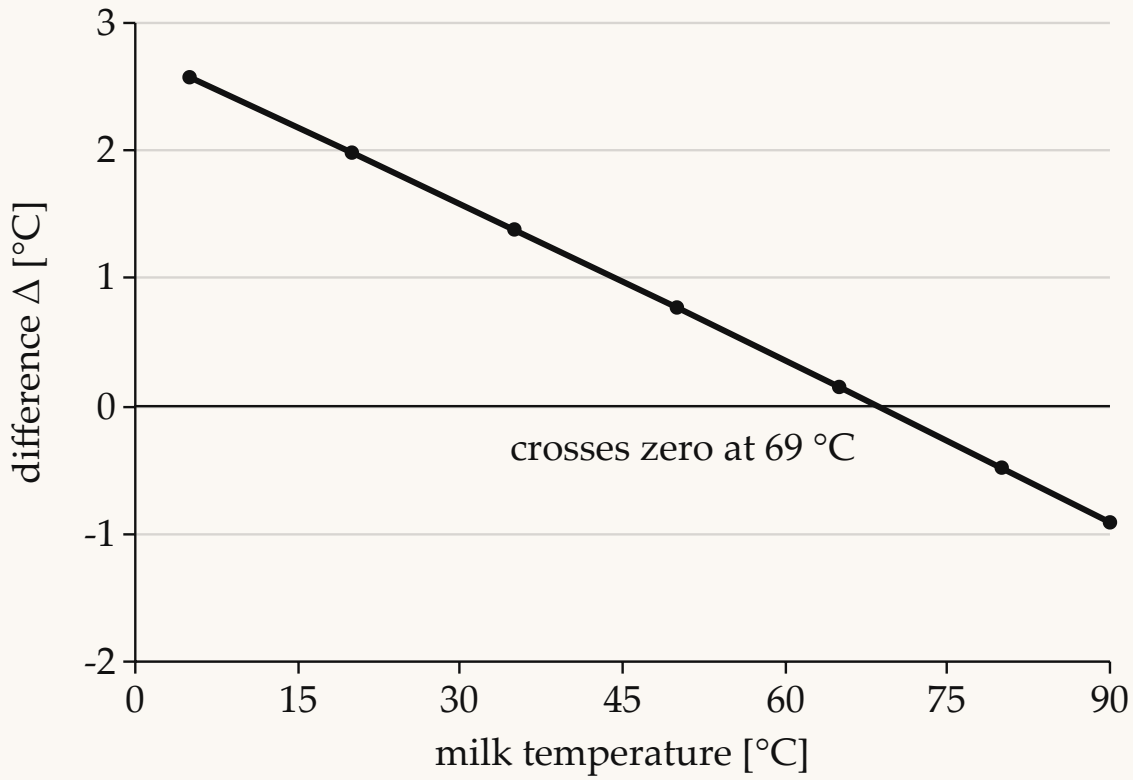
The results at this level that surprise people, each explained in full:

Milk kept hot makes adding it at the end the better choice

How it works.

1. Cold milk added immediately lowers the coffee's temperature straight away, so the drink loses heat more slowly for the whole wait. That is why adding the milk immediately is normally better.
2. Milk at the coffee's own temperature lowers nothing when it goes in. Adding it early still reduces the cooling rate slightly (more heat capacity behind almost the same surface), but now the milk's own heat leaks out for the whole wait.
3. If the milk waits on a steamer or a hob, something outside the cup keeps it hot. The coffee cools alone, and the hot milk then arrives having lost nothing. That gain is larger than the small reduction in cooling rate. In the difference formula above, hot milk makes the second term negative, and above about 69 °C it outweighs the first, so Δ is negative.

The numbers. Full model: with the milk held at 90 °C, adding it at minute ten leaves the coffee 0.9 °C hotter than adding it immediately.



How hot must the milk be before adding it at the end is better? Milk kept at a steady temperature until it is added. Δ is the temperature at minute ten with the milk added immediately minus the temperature with the milk added at minute ten. Below the crossing, adding it immediately leaves the coffee hotter; above it, adding it at the end does.

You have seen it. A barista steams the milk last and pours it into the coffee at the end.

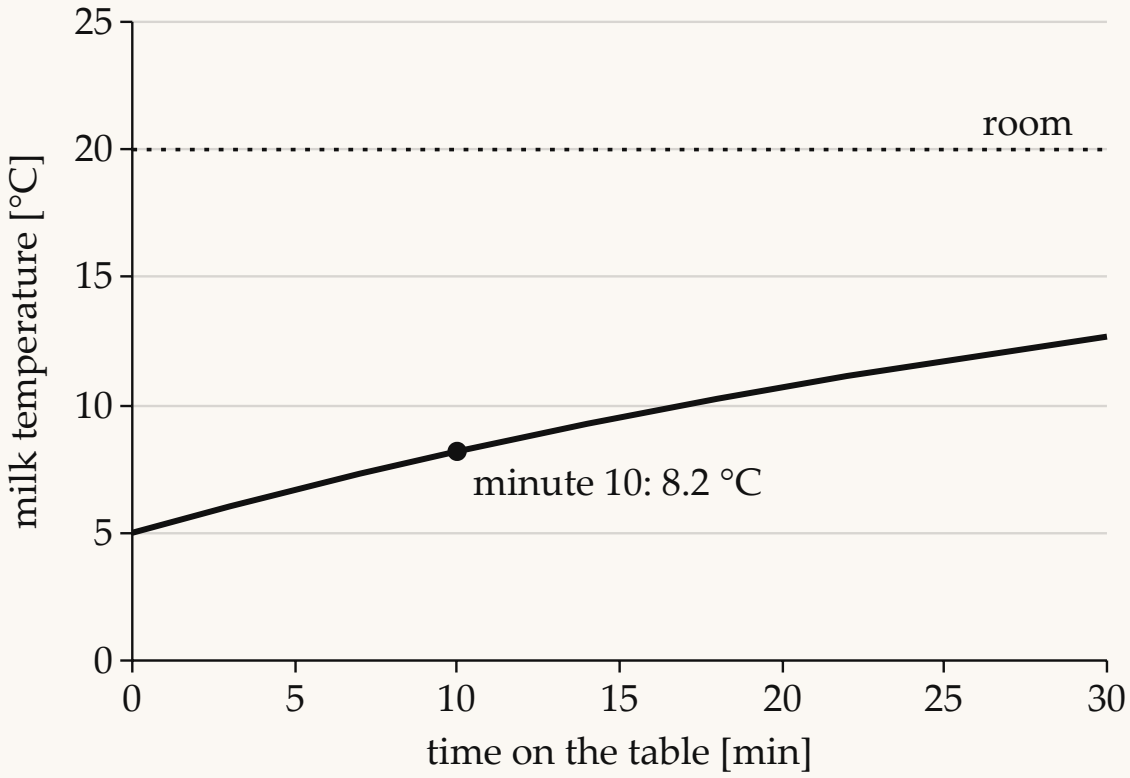
When it does not apply. Milk heated once and then left in a jug cools as well, so it is no longer kept hot. Then two cooling liquids are being compared, and the amounts and the mug decide the sign of Δ.

Milk waiting on the table reduces the advantage

How it works.

1. Fridge milk at 5 °C left in a jug warms towards the room, with a time constant of about 42 minutes.
2. By minute 10 it is at about 8 °C, so milk added at the end cools the coffee by less than fridge-cold milk would.

The numbers. Full model: Δ decreases from 2.5 to 2.3 °C; adding the milk immediately still leaves the coffee hotter. The model warms the jug with a coefficient of 8 W m^{-2} K^{-1} over 60 cm^2, an estimate for a small jug in still air.



How fast does milk on the table warm up? A jug of fridge milk left out; the dotted line is the room.

You have seen it. A jug of milk left out on the breakfast table instead of going back in the fridge.

When it does not apply. Milk kept in the fridge until the moment it is poured.

Assumptions added at this level, each with why it holds:

- **The milk waits in the fridge at 5 °C** until it is poured, unless a box below says otherwise.

What changed from the level before. The milk's own temperature now enters. On the same fixed-*K* model, fridge milk increases Δ from 1.2 to 1.7 °C; milk kept hotter than about 69 °C makes Δ negative (full model).

Level 3: where the heat actually goes

The follow-up. *"Is convection the only way the coffee loses heat?"*

The loss budget

Heat leaves a hot drink four ways. Here is each, estimated for our mug, with its working.

1. Convection from the top. Air next to the hot surface warms, rises and is replaced. For a warm surface facing up, the standard correlation (see the assumptions at the end of this level) gives a coefficient of about $h \approx 10.4$ W m^{-2} K^{-1} at 90 °C (typical range 5 to 10 W m^{-2} K^{-1} for still air; ours is at the top of the range because the gap is 70 °C). With $A_{\text{top}} = 50.3$ cm² and a 70 °C difference, $hA_{\text{top}}\Delta T \approx 3.7$ W. The sides lose about 8 W the same way, with $h \approx 9.1$ W m^{-2} K^{-1}.

2. Radiation. Every surface radiates, and the room radiates back; the net loss is $\varepsilon\sigma(T^4 - T_r^4)$ per square metre, with temperatures in kelvin: this is the formula that is not linear in temperature, so here, unlike in the mixing of level 0, Celsius would give a wrong answer. Here $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan and Boltzmann constant, and the emissivity ε is the fraction of a perfect emitter's output that the surface gives off: 0.95 for water and 0.9 for glazed ceramic in the infrared, whatever colour they look. To compare it with convection, factor the difference of fourth powers:

$$T^4 - T_r^4 = (T - T_r)(T + T_r)(T^2 + T_r^2)$$

With each of the last two factors taken at the mean temperature, $T + T_r = 2T_{\text{mean}}$ exactly and $T^2 + T_r^2 \approx 2T_{\text{mean}}^2$, so

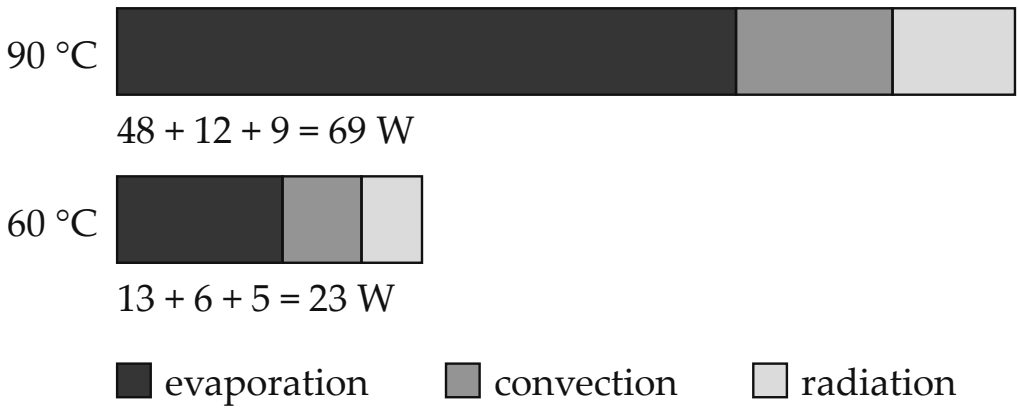
$$T^4 - T_r^4 \approx 4T_{\text{mean}}^3\,(T - T_r)$$

where T_{mean} is the mean of coffee and room, 55 °C or 328 K. Replacing the curve by a straight line like this is called linearising; at 90 °C it is 1.1 % low. Radiation then acts like a second coefficient, $h_r = 4\varepsilon\sigma T_{\text{mean}}^3 \approx 7.2$ W m^{-2} K^{-1} (with $\varepsilon = 0.9$): nearly as large as natural convection. Total radiation is about 9 W at 90 °C.

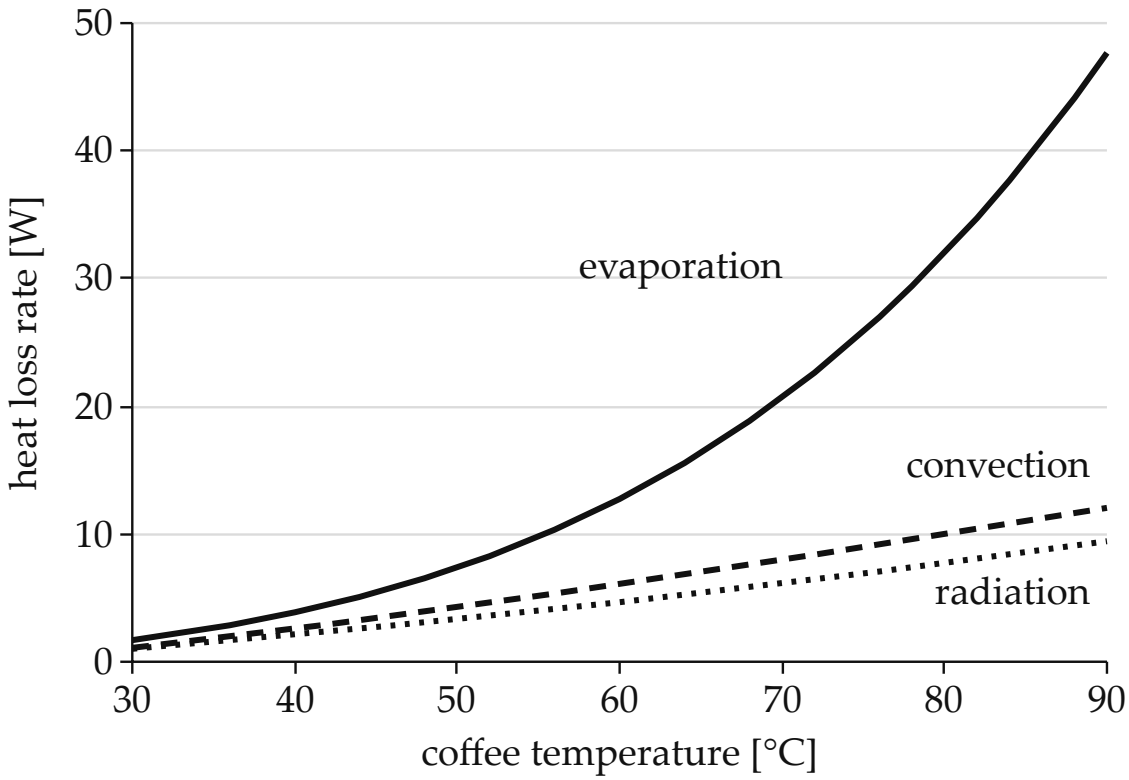
3. Evaporation. Each gram that leaves takes about 2.3 MJ kg^{-1} × 0.001 kg ≈ 2.3 kJ, enough to cool the whole mug and coffee by 1.8 °C. The vapour pressure is the part of the air's pressure due to water vapour. Air touching the water holds as much vapour as it can at the water's temperature (the saturation vapour pressure), and vapour moves from there towards the drier room. At 90 °C the vapour pressure at the surface is 70 kPa against 1.2 kPa in the room (half of the 2.3 kPa that saturated air at 20 °C would hold), so the coffee loses about 1.3 g a minute and evaporation carries about 48 W. Vapour pressure doubles about every 16 °C near 70 °C, so this route decreases quickly as the coffee cools.

4. Conduction through the wall and into the table. A ceramic wall 4 mm thick (conductivity about 1.5 W m^{-1} K^{-1}) conducts far better than the air film outside it, so the outside of the mug is nearly as hot as the coffee and the wall itself adds little resistance. A double-walled vacuum mug stops the side losses almost completely. A paper cup has thin walls and almost no heat capacity of its own. Through the base: a mug on wood loses roughly 1 W (a steady estimate: heat spreading from a disc of radius a into a solid of conductivity k meets a resistance $1 / (4ka)$; the contact disc is 7 cm across), small enough to ignore; on a stone worktop the same estimate gives about 27 W, which is not small.

Route	Heat loss at 90 °C [W]	Share [%]	Heat loss at 60 °C [W]	Share [%]
Evaporation	47.7	69	12.7	54
Convection	12.1	17	6.1	26
Radiation	9.4	14	4.7	20
Base (on wood)	about 1	left out		
Total	69.2	100	23.5	100



Where does the heat go when the coffee is hot, and when it is warm? Our mug. Evaporation is the biggest share at both temperatures but shrinks fastest; the three segments are evaporation, convection and radiation from left to right.



How does each route change as the coffee cools? Evaporation (solid) dominates when hot and falls away fastest; convection (dashed) and radiation (dotted) are close to each other all the way down.

What each route does to the time constant (heat capacity over leak per degree, at 90 °C, as in Step 7 of level 1):

- all routes: about 22 minutes, which matches everyday experience of a mug (noticeably cooler in ten minutes, below 50 °C in about 28 minutes);
- without evaporation: about 71 minutes;
- convection only: about 126 minutes;
- convection from the top only: about 7 *hours*.

So the textbook simplification, convection alone, makes the time constant about 126 minutes instead of 22. Interviews often use it anyway, because it makes the equation linear and the comparison clean; that is fine as a teaching model, and the honest thing is to say that radiation and evaporation are usually NOT negligible for a hot drink: including them shortens τ from about 126 to 22 minutes, and level 4 shows what that does to Δ.

The calculation, in order

The sums of levels 1 and 2 in the order you would do them on paper, now with the leak taken from this budget and fixed at its 90 °C value. Milk from the fridge, a wait of $t = 10$ minutes.

1. **The leak, from the loss budget.** Convection coefficients from the correlations ($h \approx 10.4$ W m^{-2} K^{-1} on top, 9.1 on the side), the radiation coefficient $h_r \approx 7.2$ W m^{-2} K^{-1}, and evaporation from the vapour pressures: $Q \approx 69$ W in all at 90 °C.
2. **Heat capacities.** $C = 1045 + 255 = 1300$ J K^{-1} for coffee and mug; $c = 121$ J K^{-1} for the milk.
3. **Leaks.** $K \approx 0.99$ W K^{-1} from the loss budget. With the milk in, the level rises and wets more wall: $K' \approx 1.01$ W K^{-1} (2.3 % more).
4. **Time constants.** $\tau = C / K \approx 22$ minutes for black coffee; $\tau' = (C + c) / K' \approx 23$ minutes with the milk in.
5. **Milk added at the end.** Cool the black coffee for t: $T_r + u_0\, e^{-t/\tau} \approx 64.4$ °C. Then mix: $(C\,T + c\,T_m) / (C + c) \approx 59.3$ °C.
6. **Milk added immediately.** Mix first: 82.8 °C. Then cool the mixture for t with τ': 61.0 °C.
7. **Compare.** Adding the milk immediately leaves the coffee 1.7 °C hotter. Check the sign against the condition of level 1: the milk added 9.3 % of heat capacity and 2.3 % of leak, so Δ must be positive.

The results at this level that surprise people, each explained in full:

Evaporation, not convection, is the largest heat loss from hot coffee

How it works.

1. Molecules escape from the surface at a rate set by the difference between the vapour pressure at the surface and in the room.
2. Vapour pressure increases steeply with temperature: 70 kPa at 90 °C against 1.2 kPa in the room, doubling about every 16 °C near 70 °C.
3. Each gram that leaves takes about 2.3 kJ of heat, so a small mass of water carries a large amount of heat.

The numbers. At 90 °C evaporation carries 48 W of the 69 W total (69 %), against 12 W of convection and 9 W of radiation: about 1.3 g of water a minute. It stays the largest of the

three all the way down to 30 °C, but its share falls below half at about 52 °C.

You have seen it. The steam over a hot drink is evaporated water condensing again in the cooler air. Blowing on soup sweeps that moist air away, so more can evaporate.

When it does not apply. With a lid there is almost no evaporation. Below about 52 °C convection and radiation together carry more heat than evaporation does.

Radiation is nearly as large as convection

How it works.

1. Every surface gives off infrared radiation, and the room's walls send radiation back; the net loss is the difference.
2. At these temperatures the difference behaves like a second convection coefficient, $h_r = 4\varepsilon\sigma T_{\text{mean}}^3$ (see the loss budget).
3. Still air carries heat away slowly, so natural convection is weak too, and the two end up of similar size.

The numbers. At 90 °C, $h_r \approx 7.2$ against $h \approx 10.4$ W m^{-2} K^{-1} on top and 9.1 on the side; radiation carries 9 W against convection's 12 W.

You have seen it. Standing near a cold window feels chilly even in warm air: your skin radiates to the cold glass, and the glass sends little back.

When it does not apply. A mug with the emissivity of polished metal, about 0.1, radiates far less from its sides: with the milk added immediately it would end at 67.0 °C instead of 65.5 °C. Moving air makes convection much larger than radiation.

A black mug cools barely faster than a white one

How it works.

1. A mug at 90 °C radiates in the infrared, with its peak near 8 micrometres, far beyond visible light.
2. Colour describes how a surface reflects visible light. In the infrared, glazes of every colour are nearly black, with emissivity about 0.9.
3. So the colour you see hardly changes the radiation that cools the mug.

The numbers. Raising the mug's emissivity from 0.9 to 0.95 changes the final temperature with the milk added immediately from 65.5 to 65.4 °C.

You have seen it. A white-painted radiator heats a room as well as a black one.

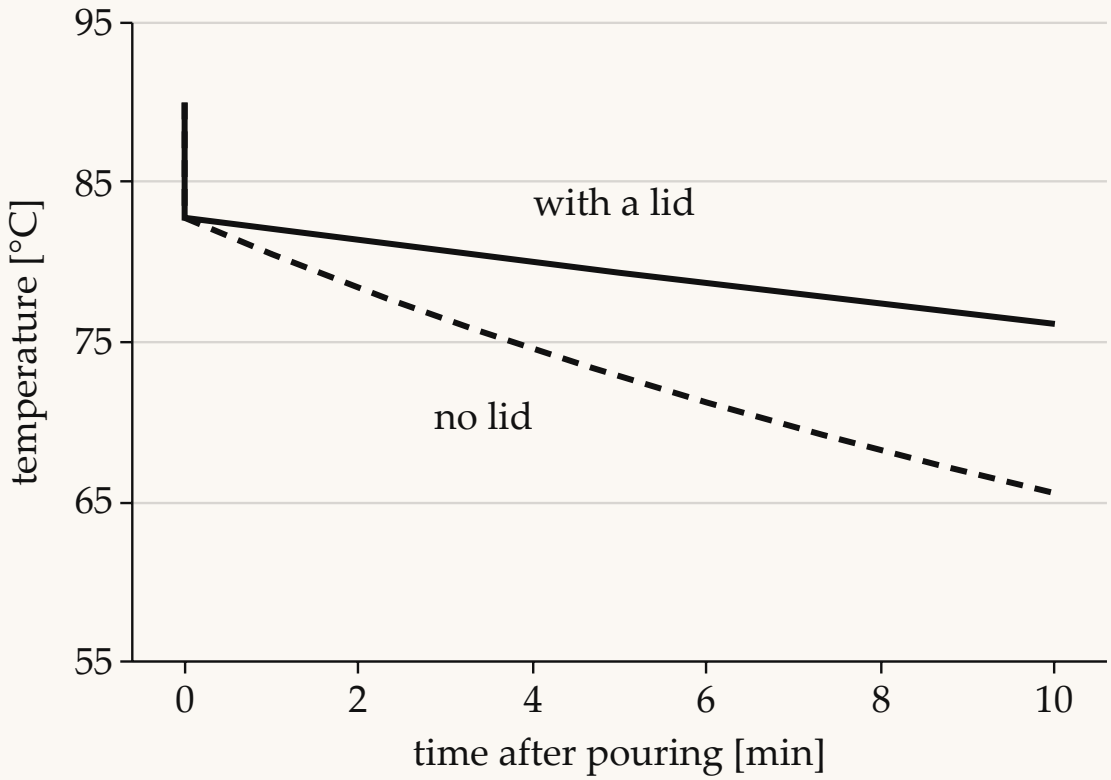
When it does not apply. In sunlight, which is mostly visible light, a black mug absorbs more than a white one and warms more. A polished metal mug, which is not black in the infrared, radiates far less. The ESAT frames this as colour and finish: a dark, matt surface absorbs and emits radiation best, a shiny, light one worst; for glazed mugs in the infrared the difference is the small one above.

A lid keeps the coffee hotter than the timing does

How it works.

1. The timing only shifts when the losses happen; it removes none of them.
2. A lid removes the largest one, evaporation (69 % of the loss at 90 °C), and, in the model, the convection from the top as well.

The numbers. With the milk added immediately, a lid keeps the coffee 10.6 °C hotter than no lid, about 4 times Δ. In the model the lid stops evaporation and the convection from the top; the top still radiates, and the lid's own heat capacity is left out.



How much does a lid save? Milk added immediately in both; with a lid (solid) against no lid (dashed). The lid stops evaporation and the convection from the top.

You have seen it. Takeaway cups have lids, and a saucer on top of a cup keeps a drink hotter.

When it does not apply. If you drink straight away there is nothing for a lid to save; its gain builds up over the wait.

Assumptions added at this level, each with why it holds:

- **Natural convection follows the standard correlations, for every mug shape.** The top is treated as a warm surface facing up, $\mathrm{Nu} = 0.54\,\mathrm{Ra}^{1/4}$, and the side as a vertical wall, $\mathrm{Nu} = 0.59\,\mathrm{Ra}^{1/4}$, where the Rayleigh number Ra measures how strongly warm air rises. In symbols, the Nusselt number $\mathrm{Nu} = hL\,/\,k_{\mathrm{air}}$ compares the coefficient with what still air would conduct across a length L ($k_{\mathrm{air}} = 0.028$ W m^{-1} K^{-1}), and $\mathrm{Ra} = g\,\Delta T\,L^3\,/\,(T_{\mathrm{film}}\,\nu\,\alpha)$, with g gravity, ν and α the air's viscosity and thermal

diffusivity, and T_{film} the mean of surface and room in kelvin. L is the area over the perimeter for the top (2.0 cm in our mug) and the wetted height for the side. Both hold over the ranges of Rayleigh number given in the standard heat transfer tables, for still air, and our surfaces fall inside them. In our mug the top stays inside its tested range (10^4 to 10^7) down to about 38 °C, which covers the whole ten-minute wait, and every mug's side wall stays inside its range. A narrowing or flaring mouth sits a little below the rim, so the air moves differently over it, and the correlation becomes an estimate there. That is safe for the question asked, because Δ depends on how much the leak increases when the milk goes in compared with the heat capacity, not on the coefficient's exact size. Halve or double every air-side coefficient and every one of the seven mugs keeps the sign of its Δ: in the narrowing cup Δ is 2.2 and 4.0 °C (coefficients halved, doubled), and in the flared cup adding the milk at the end stays better by 0.4 and 0.6 °C. Where it is stretched: the tallest, thinnest glass's surface reaches a Rayleigh number of about 5,400, just below the tested range, and a cup left for hours near room temperature is far below it, so the end temperature of that cup (level 7) is an estimate.

- **Radiation with emissivities of 0.95 for the water and 0.9 for glazed ceramic**, exchanging with room surfaces at room temperature. Water and glazes are nearly black in the infrared, whatever colour they look. It would fail for a polished metal mug, whose sides radiate far less.
- **Evaporation follows from convection by the heat and mass transfer analogy.** The same thin film of air carries heat and vapour away, so the convection coefficient sets the evaporation rate, with the room at 50 % humidity. This is the least certain route on the page, so we tested it: with evaporation switched off, halved or doubled, Δ in our mug is 0.5, 1.8 and 3.3 °C, positive in every case.
- **The base is left out.** On a wooden table it would lose about 1 W, small next to the other routes; on a stone worktop it would not be small (see the conduction paragraph above).
- **Still air**, until level 6.

What changed from the level before. The leak is no longer one number: level 1's $K \approx 0.99$ W K⁻¹ is now four routes, with evaporation 69 % of it at 90 °C. Convection alone would give $\tau \approx 126$ minutes instead of 22. With the same K the hand calculation still gives $\Delta = 1.7$ °C; what this level adds is the reason level 4 is needed: evaporation, the largest route, decreases fastest as the coffee cools.

Level 4: the leak per degree is not constant

The follow-up. *"Does the coffee really lose the same fraction of its excess every minute?"*

Step 8, what changes when the loss is not proportional to the gap. Steps 5 and 6 of level 1 work for any loss that depends only on the temperature: separating the variables gives the time to cool from T_0 to T as

$$t = C \int_{T}^{T_0} \frac{dT'}{Q(T')}$$

What changes is whether the integral can be done by hand.

- **A temperature-dependent h (convection alone).** In the correlations $\text{Nu} \propto \text{Ra}^{1/4}$ and $\text{Ra} \propto \Delta T$, so the coefficient increases like the gap to the power one quarter: $h = h_0 \, (u / u_0)^{1/4}$, with h_0 its value at the start. Then $Q = hAu = b\,u^{5/4}$ with $b = h_0 A \, u_0^{-1/4}$, about 0.06 in watts per kelvin to the power 5/4 for our mug's convection. The energy balance becomes $C\,du/dt = -b\,u^{5/4}$. Separate, as in Step 5: $u^{-5/4}\,du = -(b/C)\,dt$. Integrate with the same limits as Step 6, using $\int u^{-5/4}\,du = -4\,u^{-1/4}$:

$$-4\left(u^{-1/4}-u_0^{-1/4}\right)=-\frac{b}{C}t$$

Solve for u:

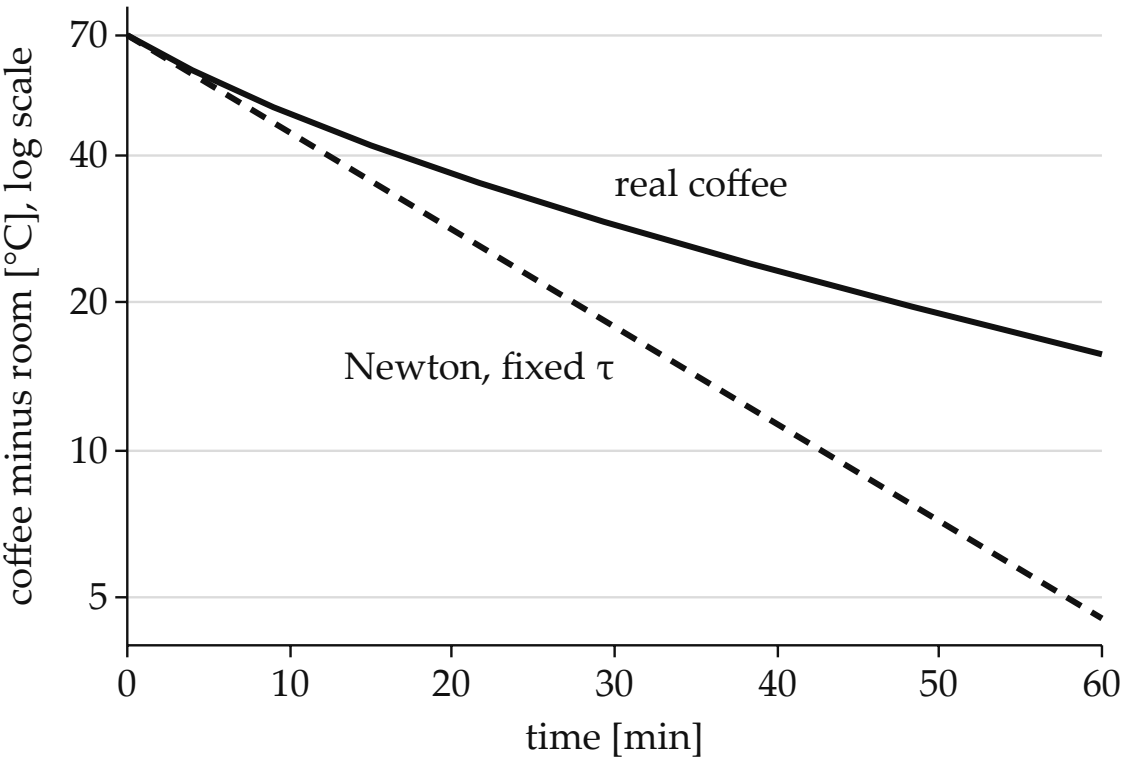
$$u=u_0\left(1+\frac{b\,u_0^{1/4}}{4C}t\right)^{-4}$$

A power law, not an exponential. For small t it starts like the exponential with $\tau_0 = C\,/\,(b\,u_0^{1/4})$, then decreases more slowly, because the leak per degree decreases as the gap decreases.

- **Radiation.** The net loss $\varepsilon\sigma A\left(T^4-T_r^4\right)$ needs kelvin. The loss budget of level 3 linearises it as $h_r = 4\varepsilon\sigma T_{\text{mean}}^3$ and adds it to K, which keeps the equation linear and the answer exponential. Kept exact, it still separates, and $\int dT\,/\,(T^4-T_r^4)$ can be done by partial fractions (logarithms and an arctangent); the linear version is within the accuracy of everything else here.
- **Evaporation.** It follows the vapour pressure, which doubles about every 16 °C near 70 °C. Written out, $Q_e = h_m A_{\text{top}} L_v\left[\rho_s(T)-\phi\,\rho_s(T_r)\right]$: h_m is the mass transfer coefficient (the convection coefficient divided by a property of air, by the analogy in the assumptions of level 3), L_v the latent heat, ρ_s the density of water vapour in saturated air and ϕ the room's humidity, 50 %. Because ρ_s increases steeply with T, $Q(T)$ has no simple form and the integral has no closed form.

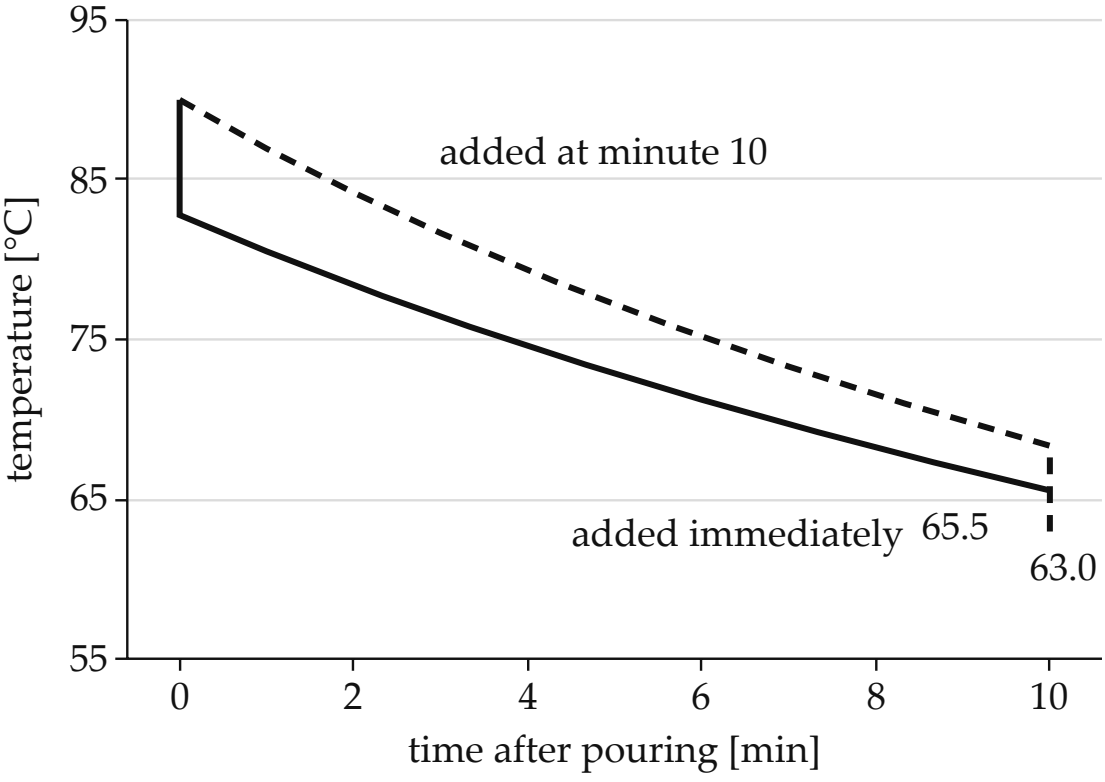
So the full model does not integrate by hand. It steps the equation forward in time: from the temperature now, estimate the slope $-Q\,/\,C$ four times across a 2 second step: at the start, twice at the middle (each time using the previous estimate to get there) and at the end. Move on by their weighted average, with weights 1, 2, 2 and 1 over 6, and repeat. This is the classical fourth order method of Runge and Kutta. The local time constant $\tau(T) = C\,u\,/\,Q(T)$ still describes how fast it is cooling at each moment.

Step 9, our τ is not constant. At 60 °C the leak per degree is lower and $\tau \approx 37$ minutes, against 22 minutes at 90 °C. Hot coffee loses more heat per degree than warm coffee because all three routes increase faster than the gap: evaporation steeply, radiation as T^4, and even convection, whose coefficient increases as $u^{1/4}$. Newton's law is the right first model, not the last word.

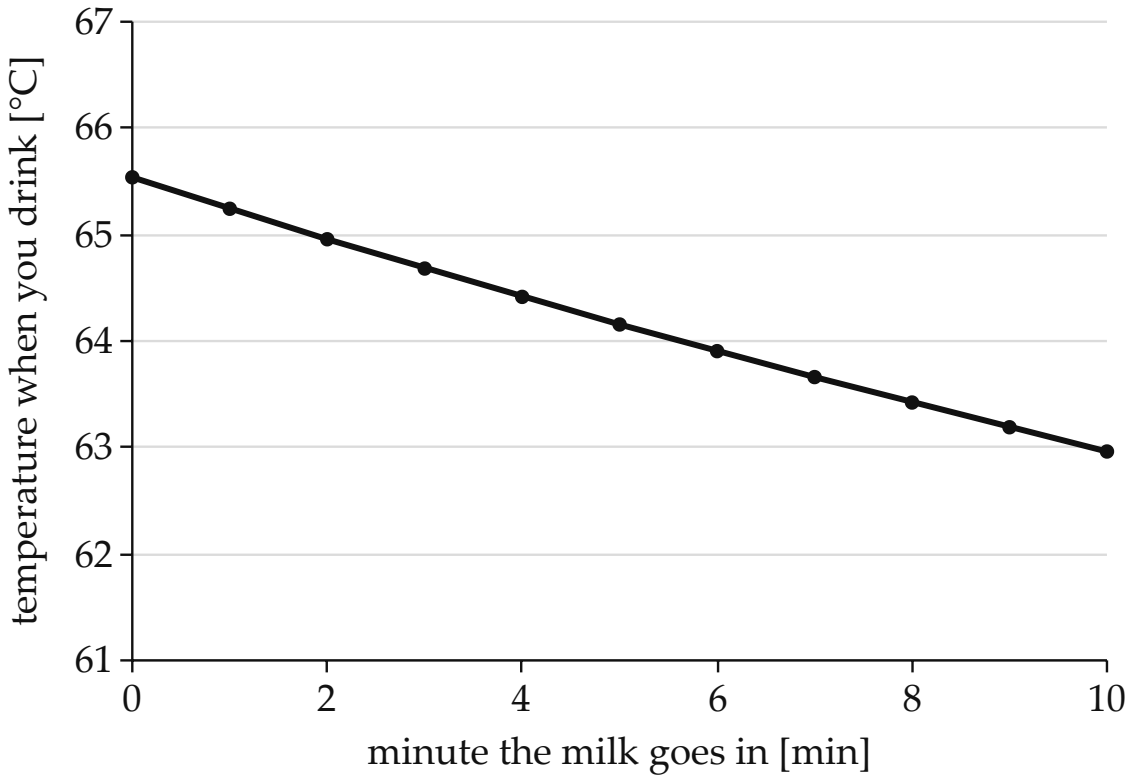


Is the cooling really exponential? Black coffee in our mug for an hour, on a logarithmic vertical scale, where an exponential is a straight line. A pure exponential with the starting time constant (22 minutes) is the dashed line; the real curve flattens as the time constant increases.

The race in a real mug. The full model solves the energy balance every 2 seconds with every route recomputed at the current temperature and level, and at the chosen minute mixes in the milk with the weighted average of level 0.



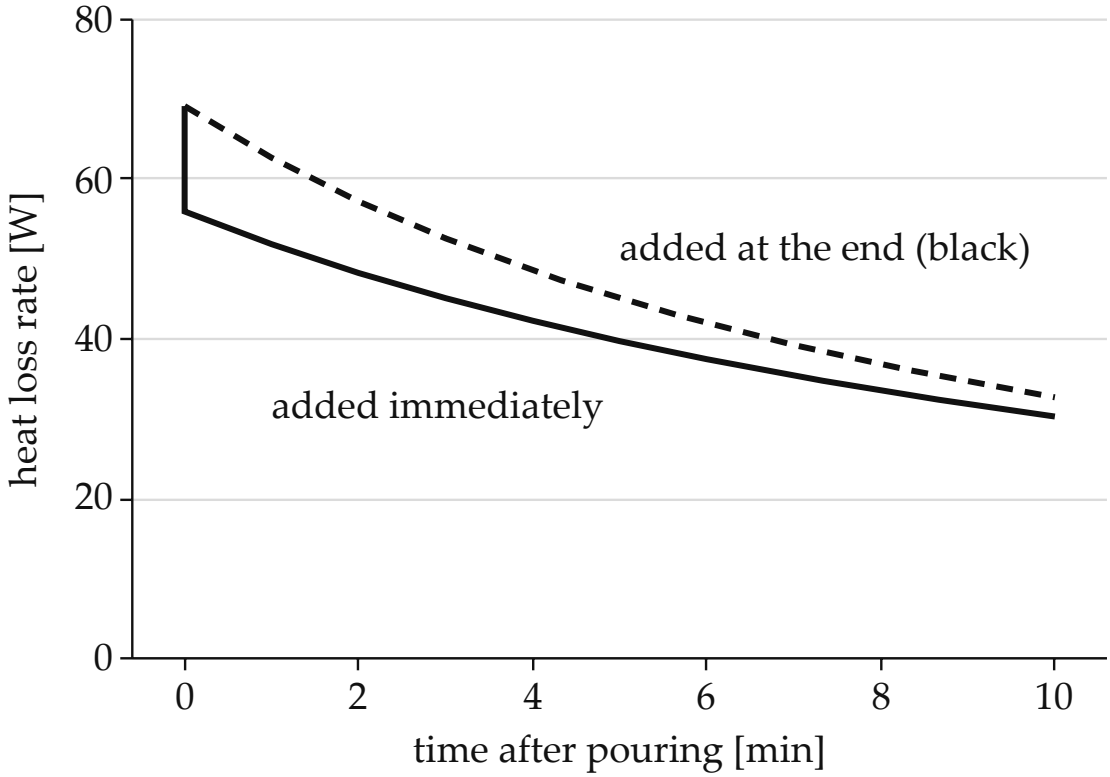
Which order leaves the coffee hotter when you drink it? Our mug with fridge milk. Milk added immediately (solid): the temperature drops by 7.2 °C when the milk goes in, then decreases more slowly than the black coffee. Milk added at minute ten (dashed): the black coffee stays hotter and loses heat faster, then drops by 5.4 °C when the milk goes in at minute ten.



Does it matter exactly when the milk goes in? Our mug, fridge milk, drunk at minute ten. Each minute of delay lowers the final temperature by about 0.3 °C, so the earliest moment is the best one.

Three causes add up to $\Delta = 2.5$ °C. With milk at room temperature only the first two act, and give 2.0 °C; fridge milk adds the third, 0.5 °C:

1. **More mass without more surface.** The milk adds 9.3 % of heat capacity behind almost the same surface.
2. **Evaporation cut early.** Adding the milk immediately lowers the coffee at once, by 5.9 °C with milk at room temperature, which lowers the vapour pressure at the surface, and evaporation with it, for all ten minutes.
3. **The extra effect of colder milk** (level 2): its deficit decays with the drink, and the colder mixture evaporates even less. Fridge milk lowers the coffee by 7 °C at once, and the vapour pressure at the surface by 24 %.



How fast is heat leaving, minute by minute? The area under each curve is the heat lost. With the milk added immediately (solid) the coffee loses heat more slowly at every moment, which is the whole answer in one picture.

Adding the milk at minute 0, 1, 2 and so on, the final temperature decreases steadily from 65.5 °C (minute 0) to 63.0 °C (minute 10): the earliest moment is the best one.

What changed from the level before. The leak per degree now decreases as the coffee cools (τ = 22 minutes at 90 °C, 37 at 60 °C). With fridge milk the full model gives 65.5 and 63.0 °C, Δ = 2.5 °C, against 1.7 °C on the fixed-K model of levels 2 and 3; with milk at room temperature, 2.0 °C against level 1's 1.2 °C. The fixed-K model has the right sign and a size about 32 % low, because it keeps the hottest, leakiest K for the whole wait.

Level 5: the shape of the mug

The follow-up. *"Does the shape of the mug matter?"*

Our seven mugs: a straight mug 8 cm across; a tall thin glass 5.5 cm and a wide cup 10.5 cm across; a cone widening at $\alpha = 15°$ from a base radius $r_0 = 3$ cm, and one narrowing at 15° from $r_0 = 5$ cm; a round bowl of radius $R = 6.5$ cm; and a straight mug 8 cm across that flares out at 55° from the coffee's surface upwards. All seven use the same convection correlations; for a mouth that narrows or flares that is an estimate, and level 3's robustness test shows every mug keeps the sign of its Δ when every air-side coefficient is halved or doubled.

The milk raises the level, so what matters is how the surface increases with volume. With coffee volume V, milk volume V_m and liquid depth z:

Cylinder of radius R: $A = \pi R^2$ whatever the level. The milk raises the level by V_m / (πR^2) (6 mm in our mug) and adds only a band of side wall.

Cone widening upwards, base radius r_0, walls at angle α to the vertical: $r(z) = r_0 + z\tan\alpha$, so

$$A(z) = \pi\,(r_0 + z\tan\alpha)^2,$$

$$V(z) = \frac{\pi}{3\tan\alpha}\left[(r_0 + z\tan\alpha)^3 - r_0^3\right]$$

Eliminate z, because the milk is given as a volume, so the surface is wanted as a function of volume: from $V(z)$, $(r_0 + z\tan\alpha)^3 = r_0^3 + 3V\tan\alpha\,/\,\pi$, and A is π times that to the power 2/3. (Another way: solve $V(z)$ for the new height numerically, which is what the model does for every mug.)

$$A(V) = \pi\left(r_0^3 + \frac{3V\tan\alpha}{\pi}\right)^{2/3}$$

Write $s(V) = r_0^3 + 3V\tan\alpha\,/\,\pi$, the cube of the surface radius. Adding milk multiplies the surface by $\left[s(V + V_m)\,/\,s(V)\right]^{2/3}$, always less than $(1 + V_m\,/\,V)^{2/3}$, the pure cone's limit: a 12 % increase in volume widens a cone's surface by at most 7.8 %, less than the 9.3 % the milk adds in heat capacity. So in our mug a smooth cone never makes Δ negative on its own (on its top surface; the wetted wall adds a little, but it loses far less heat per square centimetre).

Narrowing upwards: the same formulas with $\alpha < 0$. The surface shrinks as the level rises, so Δ is larger than in a straight mug.

Round bowl of radius R, filled to depth z: $A = \pi(2Rz - z^2)$ and $V = \pi z^2(R - z\,/\,3)$, so $dA\,/\,dV = 2(R - z)\,/\,(2Rz - z^2)$: the surface increases quickly when the bowl is shallow and stops increasing at the equator.

Tall thin glass against wide cup, same volume: the wide cup has the larger surface, so a shorter time constant. A shorter time constant makes the ten minutes a larger part of the cooling, so lowering the temperature early saves more heat and Δ is larger.

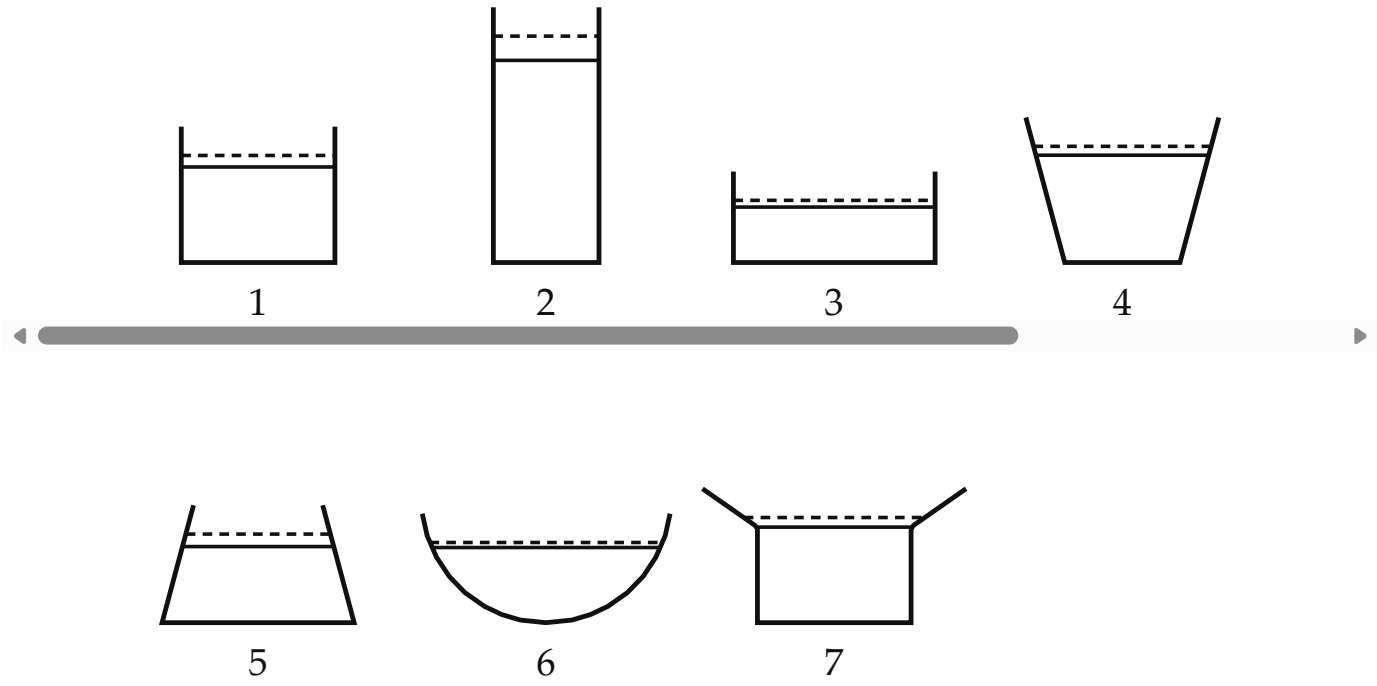
A cup that flares out exactly at the coffee's surface: all of the milk's volume goes into the widening part, and the surface increases fast. This is the shape that makes Δ negative: on the full model the sign changes at a flare of about 41 degrees from the vertical with room temperature milk and 50 degrees with fridge milk.

Each row is 250 ml of coffee with 30 ml of fridge milk added.

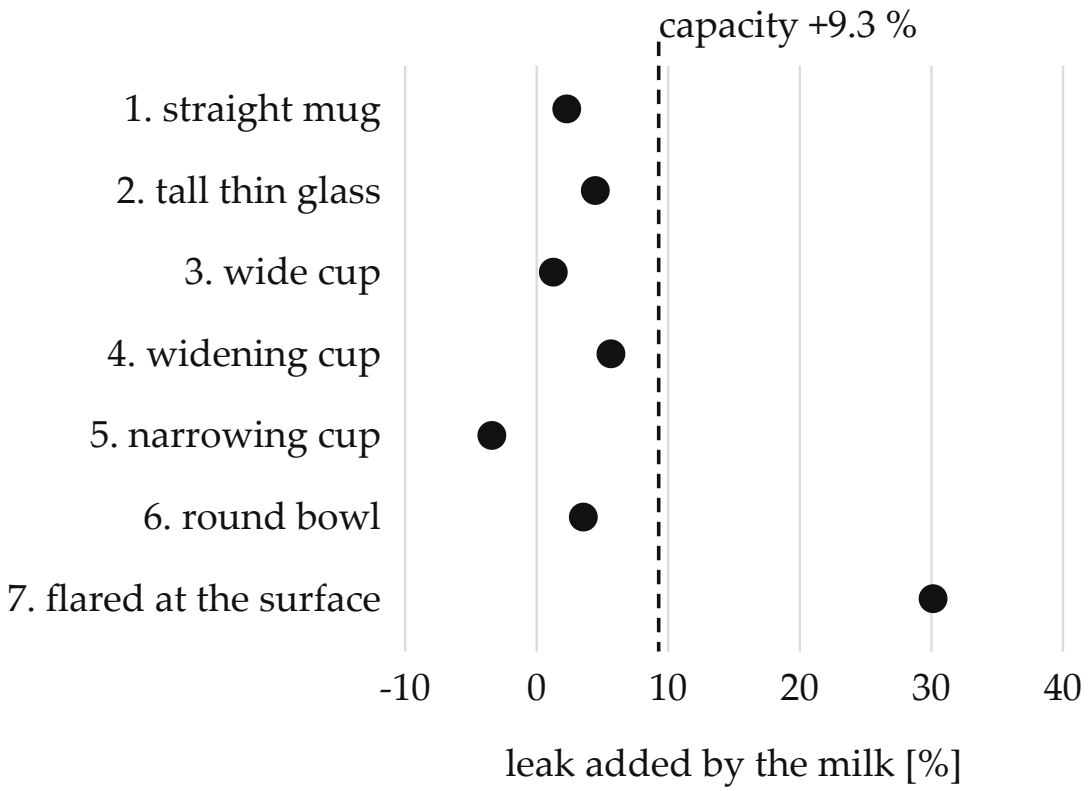
Mug	Depth [cm]	Top surface [cm²]	Area added [%]	Leak added [%]	τ [min]
1. a straight mug	5.0	`50.3 → 50.3`	`8.8`	2.3	22
2. a tall thin glass	10.5	`23.8 → 23.8`	`10.8`	4.5	31
3. a wide cup	2.9	`86.6 → 86.6`	`6.6`	1.3	15
4. a cup widening	5.6	`63.5 → 67.1`	`8.6`	5.6	18

Mug	Depth [cm]	Top surface [cm²]	Area added [%]	Leak added [%]	τ [min]
upwards					
5. a cup narrowing upwards	4.0	`48.7 → 44.5`	`7.9`	−3.4	23
6. a round bowl	3.9	`111.7 → 115.8`	`5.3`	3.5	12
7. a cup flaring out at the coffee's surface	5.0	`50.3 → 69.9`	`24.3`	30.1	22

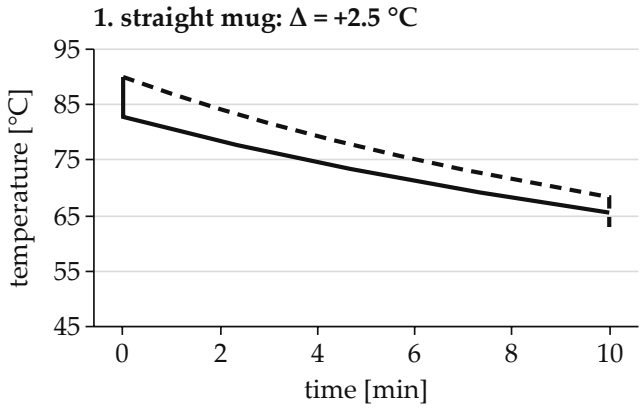
Heat capacity added is 9.3 % for every mug (the same 30 ml). Every mug is given the same ceramic, 255 J K^{-1} and a 4 mm wall, so only the shape differs. “Area added” counts the top and the wetted wall; “leak added” weights each by how much heat it actually loses, which is why the top, where evaporation happens, contributes most.

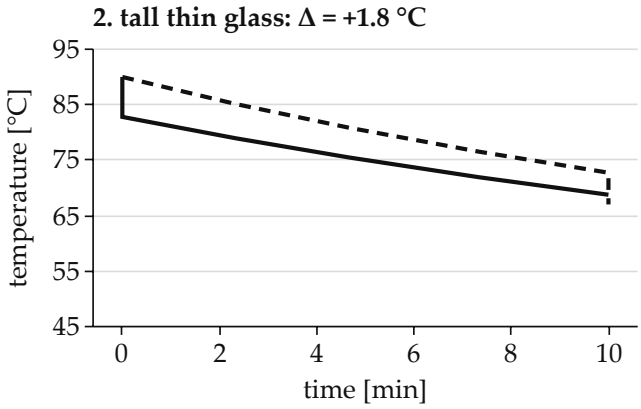


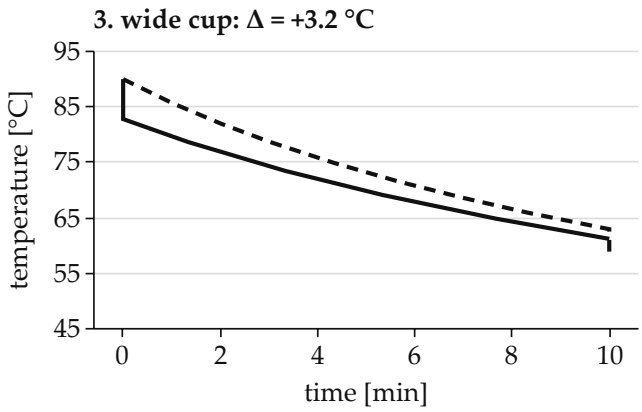
What do the seven mugs look like? Cross-sections to one scale, numbered as in the table: 1. straight mug; 2. tall thin glass; 3. wide cup; 4. widening cup; 5. narrowing cup; 6. round bowl; 7. flared at the surface. Solid line: the coffee's surface; dashed: the surface after the milk.

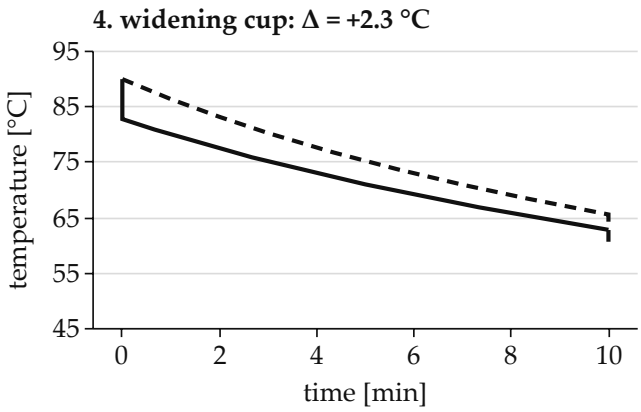


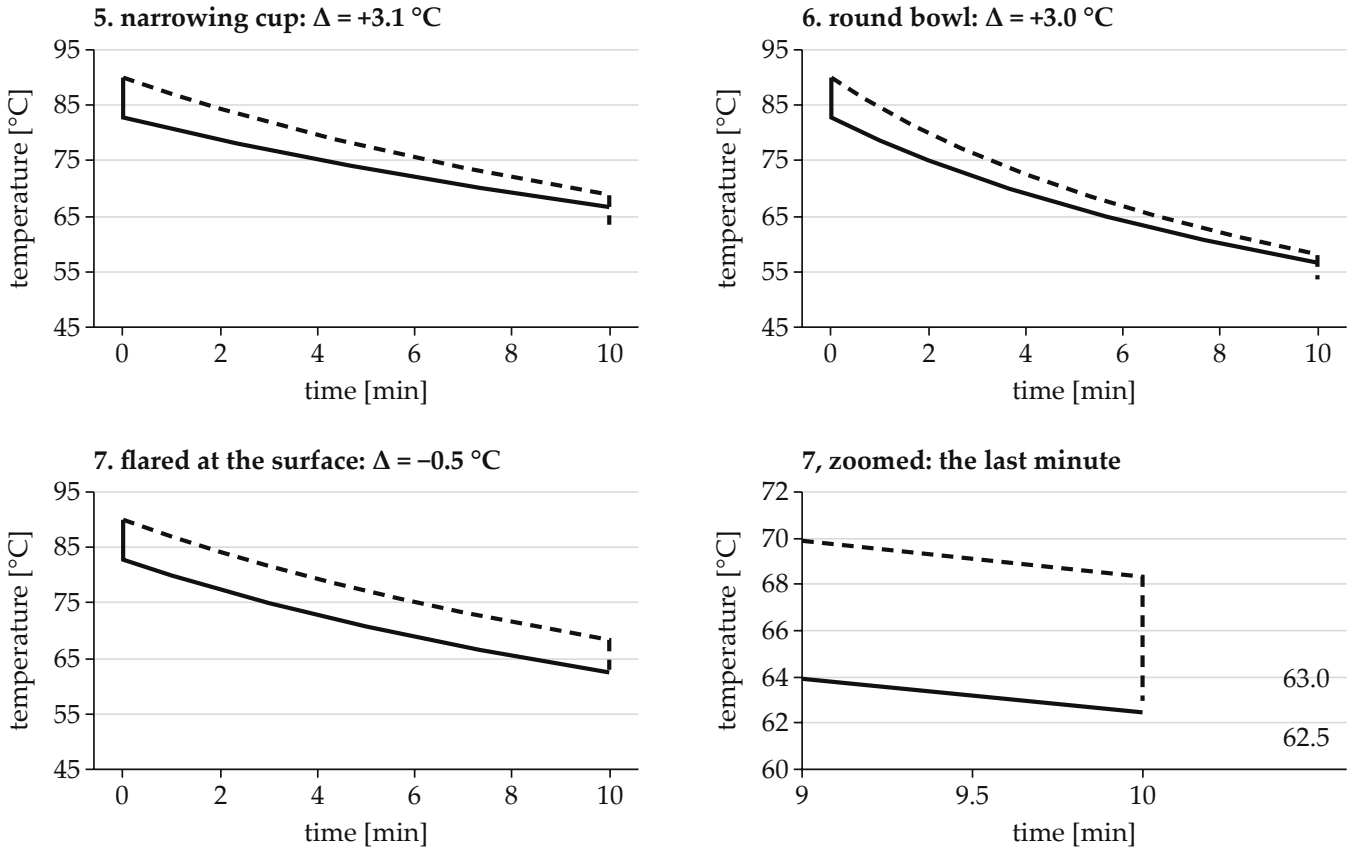
In which mugs does adding the milk immediately keep the coffee hotter, on Newton cooling alone? The milk adds 9.3 % to the heat capacity in every mug (dashed line). A mug whose leak grows by less, a dot to the left, favours adding the milk immediately; only the cup flaring at the surface lies to the right.



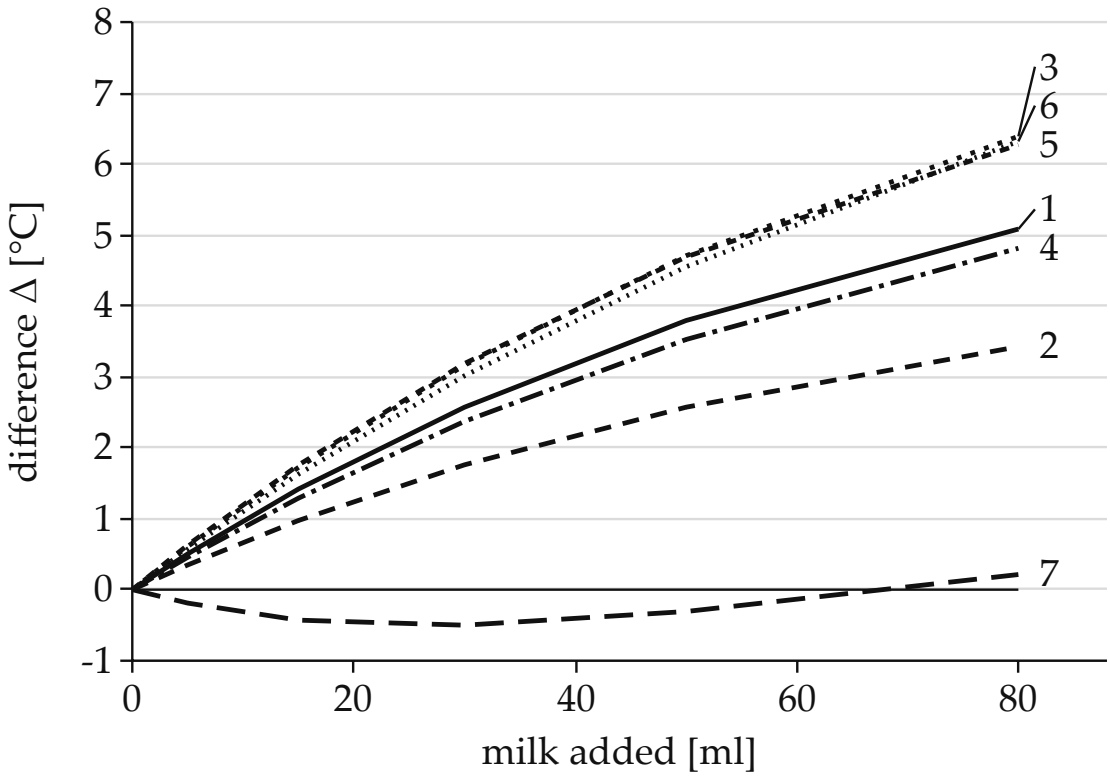








How does the mug's shape change the race? Temperature against time in each mug: milk added immediately solid, milk added at minute ten dashed, with 30 ml of fridge milk and 250 ml of coffee at 90 °C. Each title gives Δ, the final temperature with the milk added immediately minus that with the milk added at minute ten.



How much milk does it take for the order to matter? Each line is one mug, numbered as in the table. Δ is the final temperature with the milk added immediately minus that with the milk added at minute ten; positive means adding it immediately leaves the coffee hotter. With 5 ml of milk, Δ is at most 0.6 °C in any mug.

The results at this level that surprise people, each explained in full:

A cup that flares at the surface makes adding the milk at the end better

How it works.

1. Adding milk increases the drink's heat capacity by 9.3 %. In a straight mug the surface does not change, and the leak per degree increases by only 2.3 % (the milk wets a little more wall). Heat capacity increases faster than the leak, so the mixture cools more slowly and adding the milk immediately leaves the coffee hotter.
2. In a cup flaring out just above the coffee, the 30 ml of milk lifts the level into the flare. The surface increases by 39 % and the leak per degree by 30 %.
3. Now the leak increases faster than the heat capacity. Adding the milk early would make the drink lose heat faster for the whole wait, so adding it at the end leaves the coffee hotter.

The numbers. In a cup flaring at 55 degrees, adding the milk at minute ten leaves the coffee 0.5 °C hotter. The sign of Δ changes at a flare of about 41 degrees from the vertical with milk at room temperature, and 50 degrees with fridge milk. Exactly at the threshold both orders would give the same temperature, which is a boundary, not a real cup.

You have seen it. A wide cappuccino cup filled up to where it curves outward.

When it does not apply. Straight mugs, cups narrowing towards the top, and cups that widen gently: in a cup widening at 15 degrees, Δ is still +2.3 °C.

A cup narrowing at the top makes adding the milk immediately even better

How it works.

1. The milk raises the level into a narrower part of the cup, so the surface shrinks.
2. The heat capacity increases by 9.3 % while the leak per degree decreases, so the mixture keeps its heat even better.

The numbers. The surface shrinks by 8.6 %, and Δ is 3.1 °C, against 2.5 °C in the straight mug.

You have seen it. A tapered travel mug that narrows towards its lid.

When it does not apply. Once the level is in the straight or widening part of a cup, Δ returns to the straight mug's value or below.

Assumptions added at this level, each with why it holds:

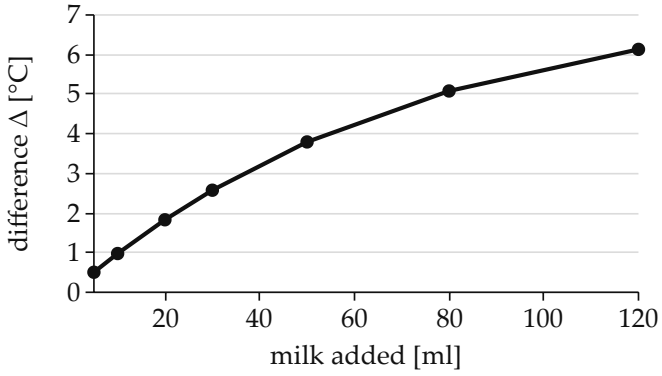
- **Every mug has the same ceramic**, 255 J K^{-1} and a 4 mm wall, so only the shape differs.

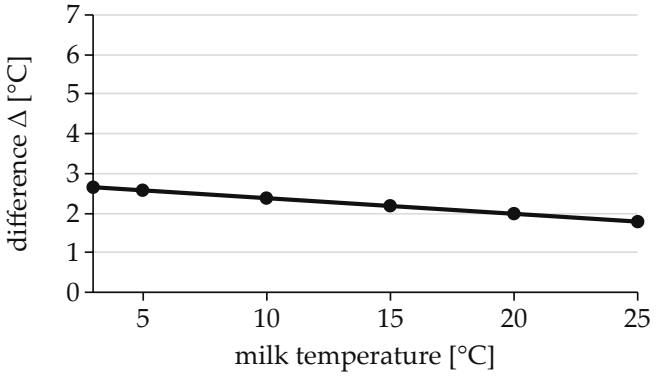
What changed from the level before. The surface can now change when the milk goes in. Against Δ = 2.5 °C in the straight mug (full model, fridge milk), Δ is between 1.8 and 3.2 °C in the five other mugs that do not flare, and −0.5 °C in the cup that flares at the surface.

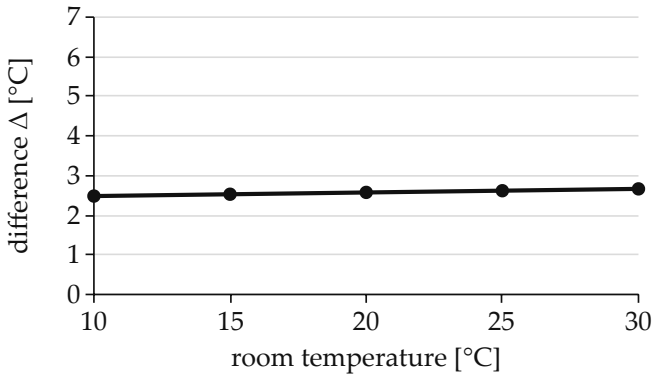
Level 6: the other variables, one at a time

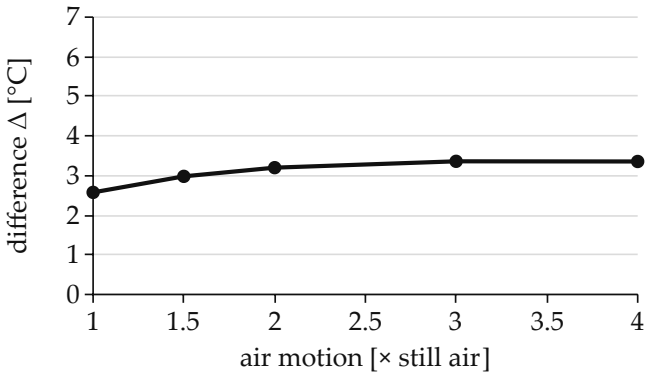
The follow-up. *"What else changes the answer, and by how much?"*

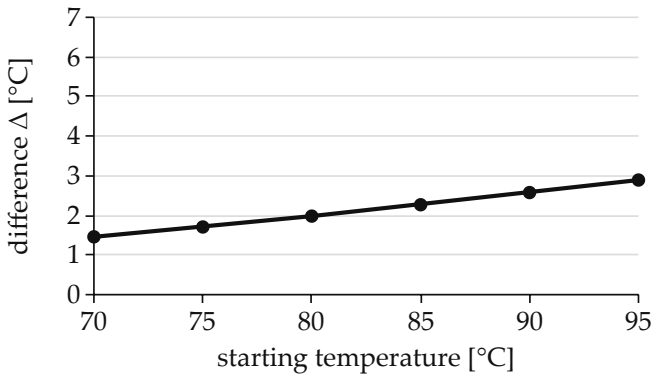
This level is a sensitivity sweep: the one complication it adds is that the base case's fixed values now move, each on its own, while the rest stay at the base case. All numbers are from the full model with fridge milk.

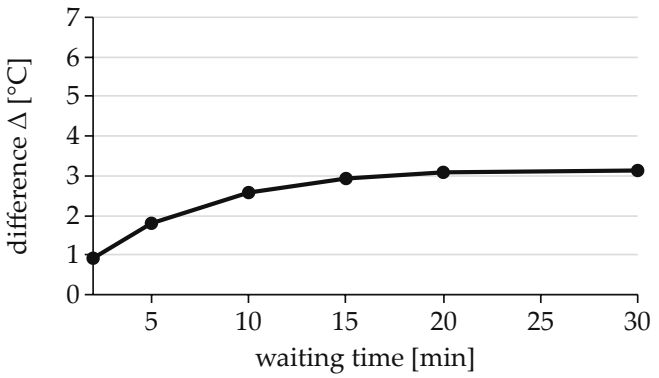


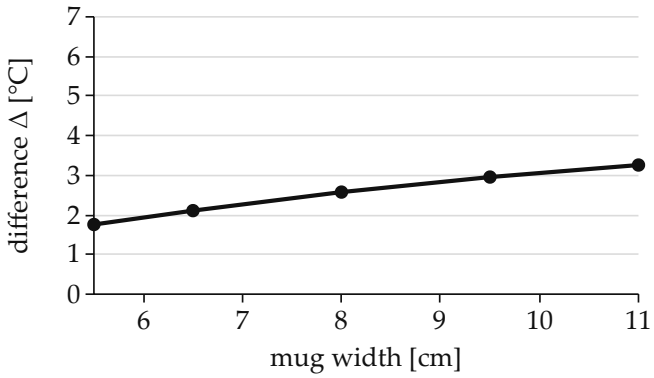




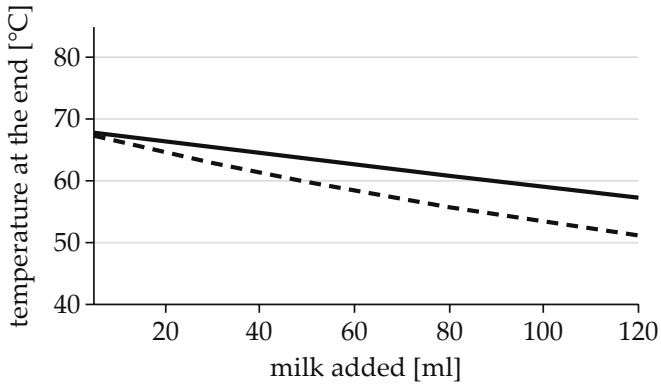


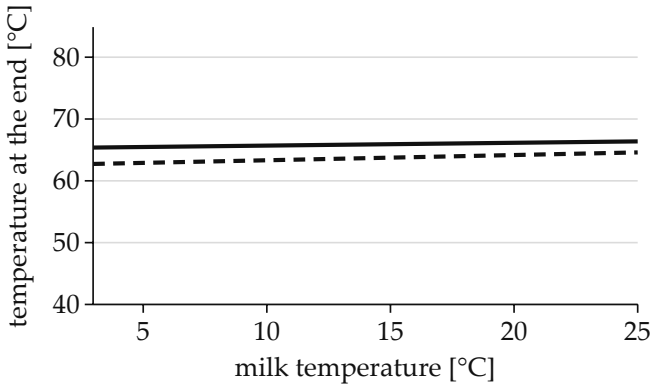


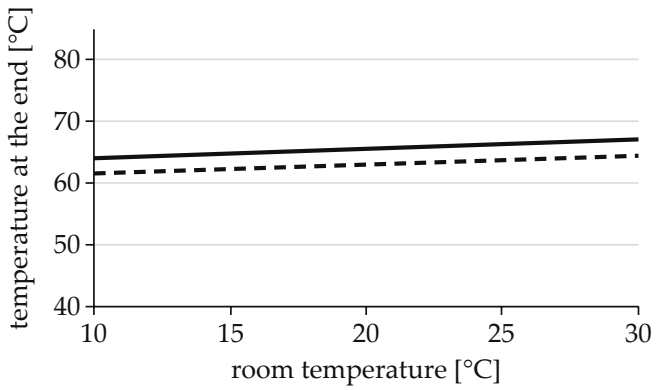


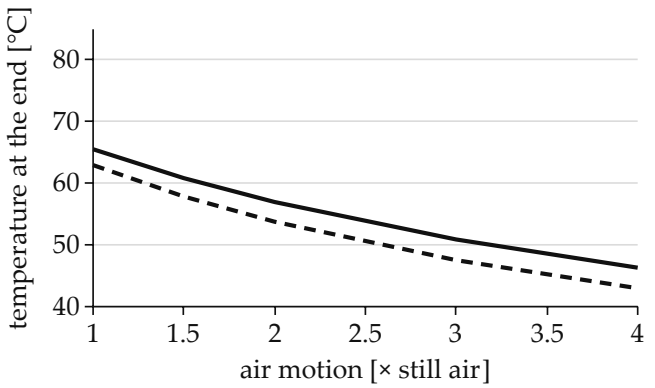


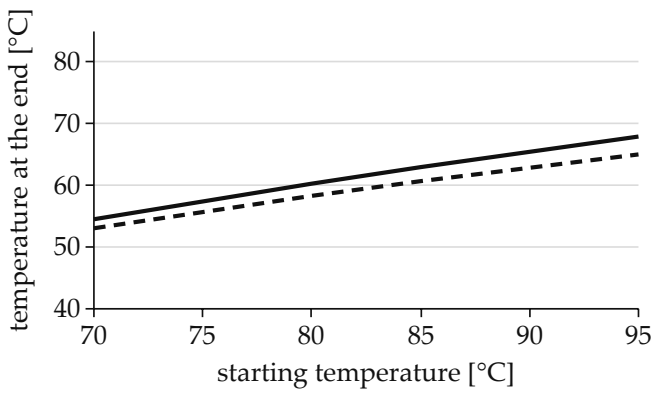
How much hotter does adding the milk immediately leave the coffee, as each variable changes? Δ is the final temperature with the milk added immediately minus that with the milk added at minute ten. One variable at a time from our base case (250 ml at 90 °C, 30 ml of fridge milk, room 20 °C, still air, ten minutes). The vertical scale is the same in every panel. The horizontal scales are in different units, so compare panels with the tornado chart below, which ranks the variables over realistic ranges.

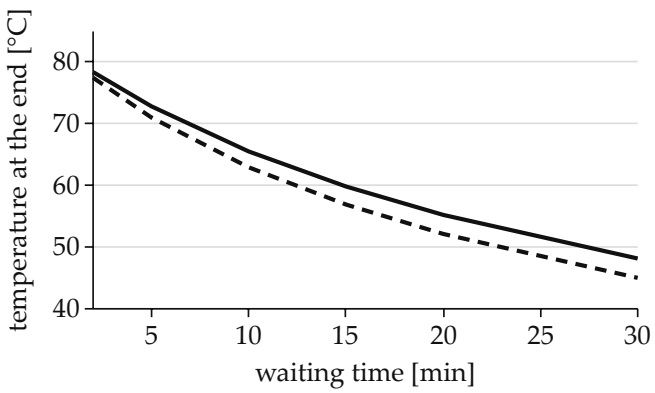


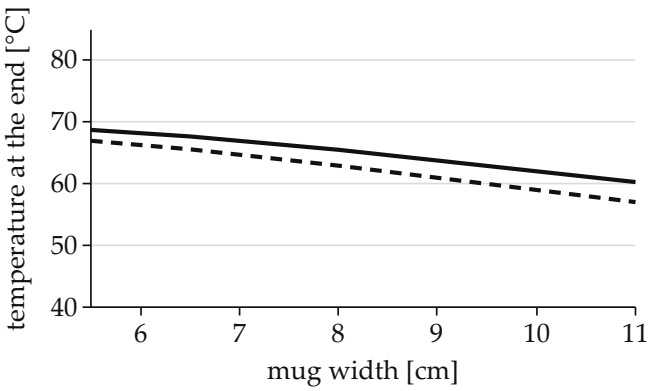








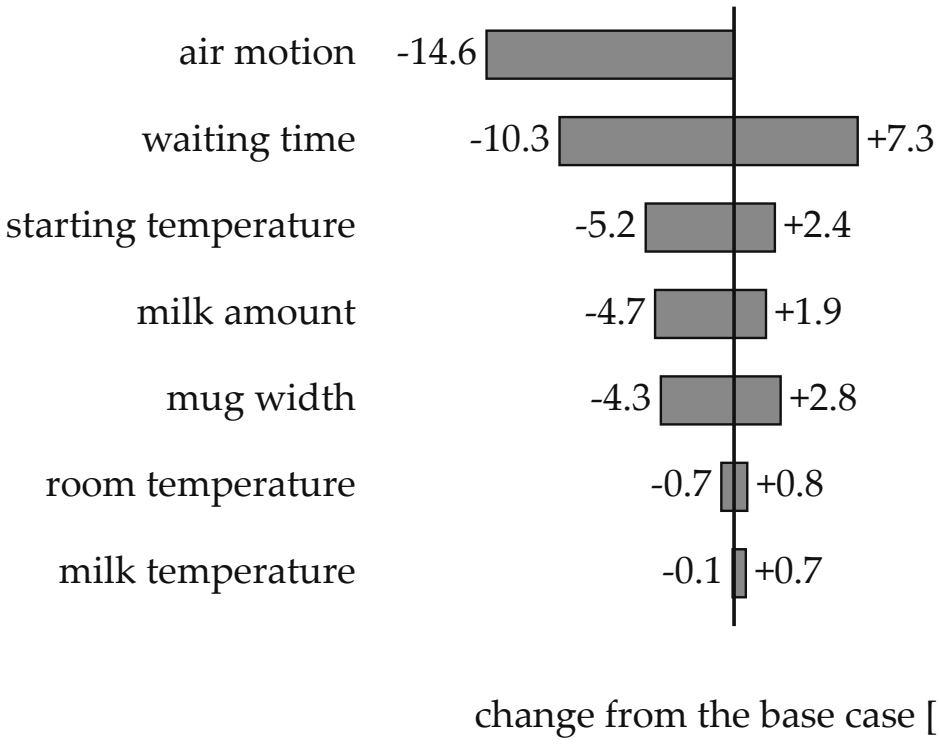




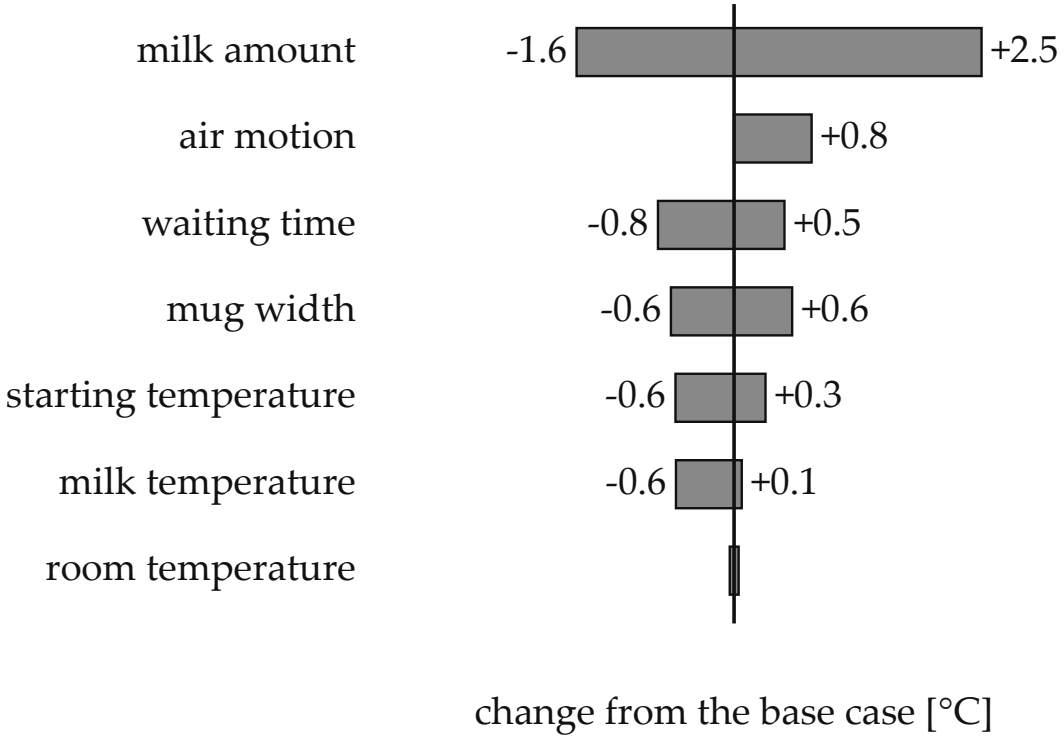
How hot is the coffee when you drink it, as each thing changes? Milk added immediately solid, milk added at minute ten dashed, same base case. Air motion and waiting time change the final temperature most (ranked in the chart below); the vertical distance between the two lines is Δ, plotted in the figure above.

How each graph changes:

- **More milk**, from 5 to 120 ml: a deeper instant drop in the race graph of level 4, Δ increasing from 0.5 to 6.1 °C, and a lower heat loss rate after mixing.
- **Warmer milk**, from 3 to 25 °C: a shallower drop, and Δ decreasing from 2.7 to 1.8 °C.
- **A warmer room**, from 10 to 30 °C: the final temperature with the milk added immediately increases from 64.1 to 67.1 °C, and Δ only from 2.5 to 2.7 °C.
- **Air motion**, from still air to 4 times the still-air coefficients (blowing on it is about 3 times; the model multiplies both the convection and the evaporation coefficients by the same factor): the final temperature with the milk added immediately decreases from 65.5 to 46.3 °C, and the log graph of level 4 becomes steeper. Δ increases from 2.5 °C in still air to 3.4 °C at 3 times and stays at 3.4 °C at 4 times, because evaporation, which adding the milk immediately reduces, increases with the air flow.
- **A hotter start**, from 70 to 95 °C: Δ increases from 1.5 to 2.9 °C, because evaporation increases more steeply at the top end.
- **A longer wait**, from 2 to 30 minutes: Δ increases from 0.9 to 3.1 °C, levelling off within about 30 minutes.
- **A wider mug** with the same coffee, from 5.5 to 11 cm across: faster cooling, and Δ increasing from 1.8 to 3.3 °C.

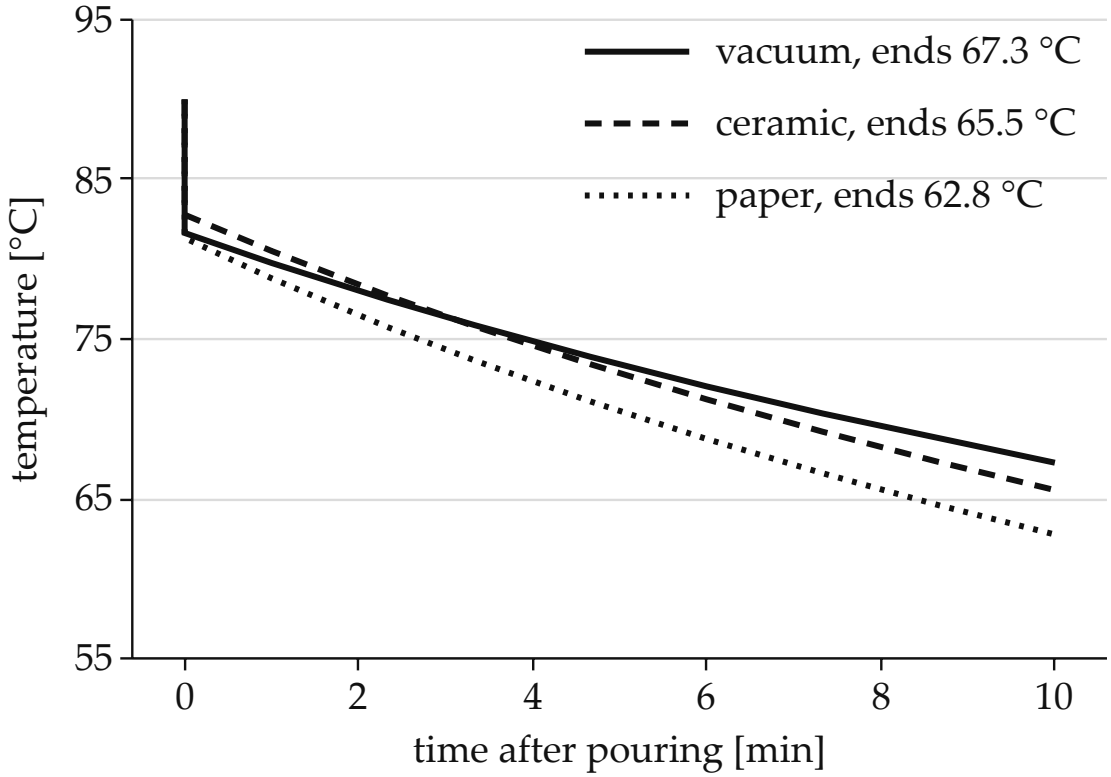


Which variable matters most for how hot the coffee is? Change in the final temperature (milk added immediately) when each variable moves across its range, all else fixed. Ranges: air motion still air to three times the coefficient; waiting time 5 to 20 minutes; starting temperature 80 to 95 °C; milk amount 10 to 80 ml; mug width 6 to 10.5 cm across; room temperature 15 to 25 °C; milk temperature 3 to 20 °C.



Which variable matters most for whether the timing matters? Change in Δ when each variable moves across its range, all else fixed. The amount of milk changes Δ the most. Ranges: milk amount 10 to 80 ml; air motion still air to three times the coefficient; waiting time 5 to 20 minutes; mug width 6 to 10.5 cm across; starting temperature 80 to 95 °C; milk temperature 3 to 20 °C; room temperature 15 to 25 °C.

The mug's material changes the final temperature too: the same comparison in a paper cup ends at 62.8 °C, in a double-walled vacuum mug at 67.3 °C (milk added immediately in both, against 65.5 °C in ceramic). In the model the paper cup has a heat capacity of 17 J K^{-1}, against the ceramic's 255; the vacuum mug has 60 J K^{-1} and loses nothing through its sides.



Does the cup's material matter? Milk added immediately in each. The vacuum mug (solid) loses almost nothing through its sides; the paper cup (dotted) holds little heat itself; the ceramic mug (dashed) is our base case.

The results at this level that surprise people, each explained in full:

A splash makes the timing almost irrelevant

How it works.

1. Adding the milk immediately keeps the coffee hotter because cooling the drink early reduces its heat loss for the whole wait.
2. The larger the milk's share of the heat capacity, the further it lowers the temperature, and the more heat it saves.

The numbers. Full model, fridge milk, straight mug: with 5 ml of milk, Δ is only 0.5 °C. It is 2.5 °C with 30 ml, and 6.1 °C with 120 ml.

You have seen it. Someone who takes just a dash of milk will not notice which order was used.

When it does not apply. With more milk the order matters more, not less, in a straight mug. In the cup that flares at the surface (level 5) Δ is negative for moderate amounts and turns positive again with a large amount.

Moving air makes the timing matter more

How it works.

1. Moving air thins the layer of warm, moist air over the cup, which multiplies convection and evaporation.
2. The coffee cools faster, so the 10 minute wait is a larger part of its cooling time.
3. Lowering the temperature early then saves more heat, so Δ increases.

The numbers. With the convection and evaporation coefficients 3 times their still-air values, the coffee with the milk added immediately ends at 50.9 °C, against 65.5 °C in still air, and Δ increases from 2.5 to 3.4 °C.

You have seen it. A desk fan, or drinking outside on a windy day.

When it does not apply. A few puffs over the surface change Δ by much less: the effect needs air moving over the cup for the whole wait.

Assumptions added at this level, each with why it holds:

- **One variable at a time**, the others held at the base case; the ranking charts use realistic ranges, listed in their captions.

What changed from the level before. Level 5's base case had every variable fixed (Δ = 2.5 °C). Moved one at a time over realistic ranges, the amount of milk changes Δ the most (0.5 to 6.1 °C), and air motion and the waiting time change the final temperature the most (down to 46.3 °C and 48.2 °C).

Level 7: the cup left for hours

The follow-up. *"What happens if nobody drinks it?"*

On Newton's law the coffee approaches the room and never goes below it. With evaporation it does go below: this is the most complete case on the page, because every route of level 3 and the humidity of the room decide where it ends.

The result at this level that surprises people, explained in full:

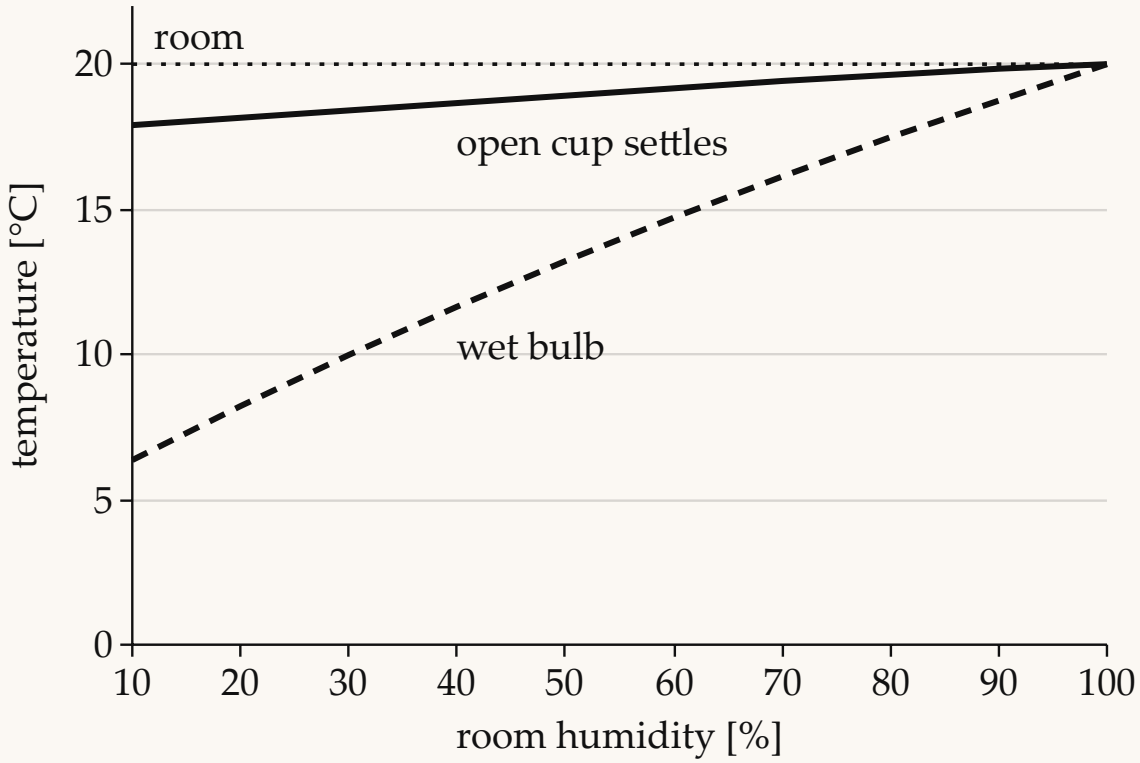
An open cup left for hours ends below room temperature

How it works.

1. Water at the surface keeps evaporating as long as the vapour pressure at the surface is higher than the room's. At 19 °C the surface holds 2.2 kPa; the room, at 50 % humidity, holds 1.2 kPa. So evaporation carries on even after the coffee has cooled to the room's temperature.
2. Each gram that evaporates takes about 2.5 kJ with it (a little more than at 90 °C, because the latent heat increases as the temperature decreases), so the coffee keeps losing heat and its temperature decreases below the room's.

3. Once it is colder than the room, the room heats it back: through the mug's side wall, by radiation from the room's surfaces, and from the air above the coffee.
4. The temperature settles where the heat coming in equals the heat evaporation takes away. In our mug that is about 19 °C, 1 °C below the 20 °C room.

The numbers. At the end temperature only 0.17 W flows each way: about 0.25 g of water evaporates an hour, and 77 % of the heat coming back in arrives through the side wall. It takes a long time to get there: a cup with the milk added immediately is within 1 °C of the room after about 3.5 hours, and the last approach has a time constant of about 2.5 hours, so it is an overnight effect. **Why it stops short of the wet-bulb temperature** (about 13 °C here): the wet bulb is where evaporation balances heat from the surrounding air alone, and a wet-bulb thermometer gets close to it because its wet wick is thin and sits in moving air, so little else warms it. A mug has a small wet surface on top of a large warm wall. The model shows each source of heat in turn: the air above the surface alone gives 13 °C, adding the room's radiation gives 16 °C, and adding the side wall gives 19 °C. Below the room the top is a cool surface facing up, with the cooled air lying still above it, so the model uses the correlation for that case, $\mathrm{Nu} = 0.27\,\mathrm{Ra}^{1/4}$, half the warm surface's coefficient. The table under the mug is left out of the model; heat through the base from a table at room temperature can only bring the end temperature closer to the room. Near room temperature the convection correlations are stretched below their tested range (see the assumptions of level 3), so 19 °C is an estimate; the balance itself puts it between the wet bulb and the room.



How far below the room does an open cup end? The cup (solid) ends between the wet bulb (dashed) and the room (dotted), about 1 °C below the room at 50 % humidity, and at the room itself in saturated air.

You have seen it. Sweat cools you in a warm room in the same way. An unglazed clay water jug keeps water below room temperature, because water seeping through the clay evaporates from its whole outside surface, a far larger wet area than the top of a mug. A wet swimsuit feels cold on a warm day.

When it does not apply. With a lid there is no evaporation, and the cup ends at room temperature. The drier the room, the lower the end temperature: 18.4 °C at 30 % humidity, 19.6 °C at 80 %, and exactly the room temperature in saturated air, where nothing can evaporate. Across the 10 to 100 % humidity of the figure the effect is small: at most 2.1 °C, too little to feel with a finger.

Assumptions added at this level, each with why it holds:

- **The cold top follows the correlation for a cool surface facing up**, $\mathrm{Nu} = 0.27\,\mathrm{Ra}^{1/4}$, because cooled air lies still above it.

What changed from the level before. Levels 1 to 6 stop at minute ten, with the coffee at 65.5 °C. Run for hours, Newton's law says the coffee approaches the room temperature and never goes below it; with evaporation, an open cup settles about 1 °C below the room, after several hours.

What an interviewer is testing at each level

- **Level 0:** a reason based on the temperature difference, and the habit of asking for or stating the missing variables.
- **Level 1:** writing an energy balance, solving it to an exponential, explaining τ, and comparing the two orders to reach $K^{'} / K < (C + c) / C$.
- **Level 2:** following one extra variable through a formula and reading the sign of each term.
- **Level 3:** estimating the watts by route, knowing when kelvin is needed, and judging which simplification is safe.
- **Level 4:** knowing what a model leaves out and what that does to the result: here, the size of Δ but not its sign.
- **Level 5:** turning a shape into geometry and finding the case that reverses the result.
- **Level 6:** ranking variables by their effect, with numbers.
- **Level 7:** following the physics past the textbook limit, and saying how certain the estimate is.

Thinking aloud (more in how to approach an interview question). Say assumptions out loud before calculating, solve the simplest model fully, check limiting cases (no milk; milk at the coffee's temperature; a mug with no leak) and units, then say what would change the answer, and in which direction. Students often worry that thinking aloud sounds unclear: it does not, and a short silence while you think is fine too.

What students often get wrong

- **The wrong driver.** Talking about solubility, or about the milk "absorbing heat", instead of the temperature difference. Heat flows because of a temperature difference (and, for evaporation, a vapour pressure difference).
- **A silent cylinder.** Assuming the mug is a cylinder without saying so. It is a reasonable assumption; say it, because the answer depends on it (level 5).
- **"Add the milk at the end, because hot coffee loses more heat and the milk then cools it down."** The first half is right and is exactly why the conclusion is wrong: the heat lost while the coffee is black is gone for good, and the milk cannot give it back.
- **Converting to kelvin for the mixing.** A weighted average gives the same result in Celsius and in kelvin (level 0); kelvin is needed for radiation's T^4 and the Rayleigh number's T_{film} (level 3).
- **"Radiation is negligible."** Near 90 °C it is about as large as convection.
- **A curve that starts slowly.** Cooling is fastest at the start, when the gap is largest: the curve leaves the axis at its steepest and flattens, never an S shape.
- **Forgetting the mug.** The ceramic is 20 % of the heat capacity.

Where students get stuck, and the step that unsticks it

- **Setting up the energy balance.** Say it in words first: rate of change of stored heat equals minus the rate heat leaves. Then $C\,dT\,/\,dt = -Q$.
- **Turning "loss proportional to the gap" into an equation.** Write $Q = K(T - T_r)$ with $K > 0$; the minus sign in the balance does the cooling.
- **Integrating** $1\,/\,u$**.** Substitute $u = T - T_r$ so $du = dT$; then $\int du\,/\,u = \ln u$, exponentiate, and the constant multiplies.
- **Reading the time constant.** Check units (J K^{-1} divided by W K^{-1} is seconds), then say what it means: the time for the gap to fall to 37 % of its starting value.

More follow-up questions an interviewer may ask

- *How long until black coffee reaches 60 °C?* About 17 minutes in our mug.
- *Does convection only happen in air?* No: the coffee convects inside the mug too, which is why treating it as one well-mixed temperature works.
- *Is a big mug or a small one better?* For the same 250 ml, a wider mug cools faster: with the milk added immediately the coffee ends at 68.8 °C in a mug 5.5 cm across and 60.3 °C in one 11 cm across, and Δ increases from 1.8 to 3.3 °C (level 6).
- *What if the milk is as hot as the coffee, or you blow on it, or nobody drinks it?* See level 2, level 6 and level 7.
- *How would you test it?* See the kitchen check below.

Check it against your kitchen

The model predicts about 3.2 °C a minute of cooling at first, black coffee at 68.3 °C after ten minutes and 36 °C after an hour, and about 8 g of water gone as vapour in the first ten minutes. Pour two identical mugs from one pot, add milk to one immediately and to the other after ten minutes, and read both with a kitchen thermometer; weigh a third mug of black coffee before and after. The least certain part of the model is evaporation (good to a factor of about two); none of the uncertainties changes the sign of Δ in a straight mug.

Where this puzzle comes from

This is a classic, argued over in kitchens and staff rooms for generations. Jearl Walker's Amateur Scientist column in *Scientific American* took the physics of a cup of coffee or tea as its subject in November 1977, and W. G. Rees and C. Viney studied the cooling of tea and coffee in theory and with household equipment in the *American Journal of Physics* (volume 56, 1988), concluding, as we do, that adding the milk at once usually keeps the drink hotter, though not in every case. The explanation, model, numbers and figures on this page are our own.